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# Influence of sample size on strength distribution of advanced ceramics

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**Abstract**

Strength distribution of advanced ceramics is mostly characterized by Weibull distribution function. The question whether the Weibull distribution always gives the best fit to strength data has been being considered in the last years. The sample size affects the reliable decision of discrimination of different distribution functions (e.g. normal, log-normal, gamma or Weibull). In this paper, 5100 experimental alumina strength data and virtual strength data generated by Monte Carlo simulations are used in order to investigate the effect of sample size on strength distribution of advanced ceramics. It is suggested that, at least 150–200 samples should be used for determination of best fitting distribution function with a statistical fallibility of 10%. Extreme Value Analysis performed with the experimental strength data showed that the Weibull distribution fits the data best and difference between the Weibull and Gumbel distributions appear at the tails.


*Keywords:* B. Failure analysis; C. Strength; D. $Al_2O_3$; Sample size; Extreme Value Statistics

## 1. Introduction

Fracture of ceramics initiates from pre-existing crack-like defects and flaws [1,2]. These flaws may be volume flaws that occur during the sintering process of a ceramic material and/or surface flaws that appear during its machining process. The strength of a ceramic is inversely proportional to the square root of the size of the most critical crack in material [3,4]. Therefore, the strength of a ceramic specimen is determined by the existing most critical crack in the volume or on the surface of the part. In 1921, Griffith [5] performed fracture experiments with glass fibers and observed that the fracture stress increases as the fiber diameter decreases. The strength depends on the stressed area or volume of a material because a larger area or volume increases the probability of existence of a critical flaw [6,7]. Here the most critical flaw does not always represent the largest flaw in the material. The size, orientation and position of a crack determine whether a crack is critical or not. The cracks are randomly distributed in the material and the position, size and orientation of the most critical flaw show scattering. Due to this scattering, strengths of ceramics vary from components to components, even if identical specimens are tested. Since the strength of ceramics is not a deterministic value, a probabilistic method is recommended for the design of advanced ceramics [8–12].

The Weibull distribution [13], based on the concept of the weakest link, is the most widely used formulation for the strength characterization of ceramics [14,15]. In most cases, it is enough to use two-parameter form of the Weibull distribution for reliable ceramic component design [16].

The question whether the Weibull distribution always gives the best fit to strength data has been investigated in literature in recent years. For example, Danzer [17] performed experiments with small specimens and observed that the Weibull theory is insufficient in estimating the strength behavior because the fracture origins are larger than the effective volume of the specimens [14]. Some possible microstructural reasons may cause deviation from the Weibull distribution. Such deviations occur in ceramics which have multi-modal flaw size distribution, R-curve behavior, subcritical crack growth and internal residual stresses [11,18]. These microstructural activities cause applied stress dependent Weibull modulus. Lu et al. [12] investigated the strength data of $Si_3N_4$, SiC and ZnO ceramics and reported that the normal distribution fits the strength data of ZnO better than the Weibull distribution. Basu et al. [19] carried out the statistical

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analysis of strength data of monolithic $ZrO_2$, $ZrO_2$–$TiB_2$ composites, glass and $Si_3N_4$ by using the probability models of the Weibull, normal, log-normal, gamma and generalized exponential distributions and reported that the gamma or log-normal distribution, in contrast to the Weibull, may more appropriately describe the measured strength data. Nohut and Lu [20] applied involving normal, log-normal and Weibull distributions to the analysis of ten strength data sets of dental ceramics with different compositions and concluded that various microstructures and compositions in the investigated dental ceramics cause their strength distributions deviated from the Weibull distribution.

Fracture experiments (e.g. three-point bending test, four-point bending test, ball-on-three balls test) are performed in order to obtain strength data of a ceramic material. The Weibull statistics is applied to strength data of advanced ceramics according to DIN V ENV843-5 standard [21] which suggests at least 30 specimens to be tested for a reasonable determination of Weibull parameters due to high production and machining costs. Danzer et al. [22] investigated the effect of number of specimens on the Weibull parameters of a silicon nitride and concluded that at least 30 specimens should be tested for the determination of Weibull parameters with a reasonable standard deviation. The discrimination of different distribution function of strength of advanced ceramics has been investigated in some articles with 30 samples and some even with less number of specimens [12]. Here the question arises whether 30 strength data is enough for a clear distinction between different distribution functions. In literature, there is no study which investigates the effect of sample size on statistical fallibility in determination of most suitable distribution type due to lack of large experimental data. In this article, 5100 experimental alumina strength data, provided by Lovro Gorjan, Hidria AET [23] is used in order to investigate the effect of sample size on Weibull parameters and on the correct determination possibility of best distribution function. The determination error is given as a function of sample size. Moreover, virtual strength data generated by Monte Carlo simulations is used in order to compare with the experimental results. By random selection of strength values as a sample size from 10 to 100, the distribution of experimental Weibull modulus will be investigated.

For a reliable design with advanced ceramics, low failure probabilities are of interest. When the low failure probabilities are expected, the Extreme Value Statistics should be performed [24]. Extreme value theorem or extreme value statistics deals with the extreme deviations from the median of probability distributions. Unlike from classical statistics which focus on the average behavior of stochastic process, extreme value theory focuses on the extreme and rare events. In the final part, the Extreme Value Analysis of the strength distribution of alumina ceramics is given.

## 2. Theoretical background

### 2.1. Distribution Functions

In the design of advanced ceramics, the design is said to be reliable when the failure probability is in the order of approximately $10^{-6}$. This means, for a reliable design, $10^6$ experiments have to be performed in order to catch the tail properties of strength distribution. Since application of $10^6$ experiments is economically not feasible, statistical models are used for the determination of strength distribution by using lower number of experiments. It is expected that the distribution function should predict so low failure probabilities by using limited number of experimental strength data in a reliable manner. In this article, four different distribution functions are used (normal, log-normal, Gamma and Weibull distributions) for fitting the strength of alumina ceramic specimens.

Normal distribution is extremely important in statistics and is often used in the natural and social sciences for real-valued random variables whose distributions are not known. The strength distribution of a brittle material without surface preparation shows a symmetrical behavior and therefore normal distribution may be a potential distribution to fit such a data [25,26]. The pdf of a normal distribution is represented as

$$p(\sigma) = \frac{1}{\sqrt{2\pi}\alpha} \exp\left[-\frac{(\sigma-\overline{\sigma})^2}{2\alpha^2}\right] \tag{1}$$

where $\overline{\sigma}$ and $\alpha^2$ are the mean and variance, respectively and can be calculated as;

$$\overline{\sigma} = \frac{1}{n}\sum_{i=1}^{n}\sigma_i \quad \text{and} \quad \alpha^2 = \frac{1}{n}\sum_{i=1}^{n}(\sigma_i-\overline{\sigma})^2 \tag{2}$$

The log-normal distribution is a distribution of a random variable whose logarithm is normally distributed. Thus, its pdf can be written as

$$p(\sigma) = \frac{1}{\alpha\sigma\sqrt{2\pi}} \exp\left[-\frac{(ln\sigma-\overline{\sigma})^2}{2\alpha^2}\right] \tag{3}$$

If a data is distributed lognormally with parameters $\overline{\sigma}$and $\alpha$, then the logarithm of the data is distributed with a mean $m = \exp(\overline{\sigma}+\alpha^2/2)$ and variance$v = \exp(2\overline{\sigma}+\alpha^2)\,[\exp(\alpha^2)-1]$.

The gamma distribution, like the log-normal distribution, is an alternative to analyze highly skewed data. The general formula for the probability density function of the gamma distribution is;

$$p(\sigma) = \frac{1}{\theta^k}\frac{1}{\Gamma(k)}\sigma^{k-1}e^{-(\sigma/\theta)} \tag{4}$$

where $k$ is the shape parameter, $\theta$ is the scale parameter and $\Gamma$ is the gamma function which has the formula

$$\Gamma(a) = \int_n^{\infty} t^{a-1}e^{-t}dt \tag{5}$$

Weibull distribution is a type of Extreme Value Distribution (EVD) and it uses Extreme Value Statistics (EVS). The problem of modeling rare events is applied in many areas where such events can have very negative dangerous consequences. Unlike from classical statistics which focus on the average behavior of stochastic process, extreme value theory focuses on the extreme and rare events. Assume that we have a vector of samples $\{X_1, X_2, , X_n\}$ from an arbitrary population. The maximum value from the sample vector is selected from the parent distribution by the operator, max $\{X_1, X_2, , X_n\}$. Dealing with the maximum values