Composites: Part A 66 (2014) 44–57

Contents lists available at ScienceDirect

Composites: Part A

journal homepage: www.elsevier.com/locate/compositesa

An elastic–plastic constitutive model for ceramic composite laminates

Varun P. Rajan *, John H. Shaw, Michael N. Rossol, Frank W. Zok

Materials Department, University of California, Santa Barbara, CA 93106, United States

ARTICLE INFO

Article history:
Received 15 April 2014
Received in revised form 19 June 2014
Accepted 21 June 2014
Available online 14 July 2014

Keywords:
A. Ceramic–matrix composites (CMCs)
B. Mechanical properties
C. Computational modeling
C. Finite element analysis

ABSTRACT

Existing phenomenological constitutive models are unable to capture the full range of behaviors of ceramic composite laminates. To ameliorate this deficiency, we propose a new model based on the deformation theory of plasticity. The predictive capabilities of the model are assessed through comparisons of computed and measured strain and displacement fields in open-hole tension tests. The agreements in the magnitude of strains and in the size and shape of shear bands that develop around a hole are very good over most of the loading history. Some discrepancies are obtained at high stresses. These are tentatively attributed to non-proportional stressing of some material elements: a feature not captured by the present model.

1. Introduction

Ceramic matrix composites (CMCs) of engineering interest exhibit some capacity for inelastic straining prior to fracture. The inelasticity can play an important role in the distributions of stresses and strains in coupons or components containing holes or notches (much like plasticity in metals). One consequence is a reduced degree of notch-sensitivity of tensile strength (relative to truly brittle materials) [1–9]. A robust capability for modeling inelasticity in CMCs would enable the design of complex engineering components and could lead to designs that are less conservative than those that would emerge from purely elastic analyses. The present article focuses on the modeling of the deformation of CMCs under multiaxial (2-D) loadings.

The primary mechanism for inelasticity in CMCs at low stresses is matrix microcracking. In notched panels of CMCs with stiff matrices (e.g. SiC/SiC, SiC/CAS), microcracking occurs first directly ahead of the notch tip and then spreads across the net section in a spatially distributed manner [10,1,5]. In contrast, microcracking in CMCs with compliant matrices (e.g. SiC/C, C/C) is manifested in shear damage bands that initiate at the notch tip and extend parallel to the axis of applied load [10,3]. The former type of composite has been termed ‘Class II’ and the latter type ‘Class III’ [11]. For both classes, fiber fracture becomes important at higher stresses, providing an additional mechanism of inelastic deformation and ultimately leading to macroscopic rupture.

Micromechanical models developed over the past three decades have provided great insights into the mechanics of matrix cracking. The onset of matrix cracking in unidirectionally-reinforced materials loaded in uniaxial tension along the fiber direction is particularly well-understood [12,13]. But such models are difficult to extend in a generic manner to laminates with other architectures and to the macro-scale. There are two reasons for this. First, principal stresses are generally oriented at an arbitrary angle with respect to the fiber axes; existing micromechanical models of even unidirectional CMCs have yet to be extended to general multiaxial in-plane loadings. Second, the mechanisms of cracking in multidirectional laminates and the associated mechanics are significantly more complex than those of the constituent unidirectional plies. Several additional damage mechanisms can arise, including crack tunneling in transverse plies, growth of partially unbridged cracks into axial plies, and delamination between plies [14,15]. It would appear that deriving macro-scale constitutive models (at scales $\gg$1 mm) directly from micromechanical models (at scales of 1 μm to 100 μm) is not currently feasible.

Unsurprisingly, existing constitutive models for CMC laminates under general multiaxial loading are largely phenomenological (not micromechanical) in nature [16]. By ‘phenomenological’, we mean that the models take as inputs macroscopic quantities such as stress–strain curves into a framework that relates stresses to strains. Two classes of models exist.

The first relies on concepts from continuum damage mechanics [17–19]. Here the state of the composite is described by internal damage variables that can be scalars, vectors, or tensors. As these damage variables evolve with applied loads, the stiffness of the composite is degraded. The damage evolution laws are calibrated

* Corresponding author. Tel.: +1 (805)893 2694.
E-mail address: varun_rajan@umail.ucsb.edu (V.P. Rajan).

http://dx.doi.org/10.1016/j.compositesa.2014.06.013
1359-835X/

using experimental data. In some models, such as that of Talreja [17], the damage variables are, in principle, measurable quantities, e.g. crack density; in others, such as that of Camus [18], they are not, and must instead be inferred from macroscopic stress–strain data. The fundamental tradeoff in all such models is between the complexity of the representation of damage and the amount of experimental data needed for calibration. For instance, the model of Talreja [17] requires evolution laws for four different damage tensors that represent crack densities, debond lengths, etc. A very extensive experimental program is thus required to calibrate the model for general multiaxial loading. Despite their complexity, it remains unclear whether the *predictive* capability of such models is actually enhanced by the large number of internal variables.

The second modeling approach dispenses with internal damage variables and instead assumes that the degradation of the composite stiffness can be related to macroscopic stress–strain functions from standard mechanical tests that elicit the important damage mechanisms. In principle, this approach should yield mathematically simpler models that can be calibrated with far fewer experimental data, since there are far fewer 'fitting constants.' Models of this type have been developed mainly for use with polymer matrix composites [20–22]. An attempt to extend the approach to CMC laminates was made by Genin and Hutchinson [2] and later expanded by Rajan and Zok [23]. For reasons elaborated upon in Section 3, however, these models are limited in applicability; that is, they presuppose a particular form of the yield/cracking surface that may be consistent with the behavior of some material systems but not others. Here we address these deficiencies and propose an alternative model that is less restrictive yet equally straightforward to implement.

The principal objective of the present study is to develop a plane-stress, phenomenological elastic–plastic model for CMC laminates that satisfies the following criteria:

1. It can be calibrated using experimental data from standard mechanical tests (e.g. tension, shear).
2. It is applicable to common CMC fiber architectures, notably cross-ply ($[0°/90°]_s$) and quasi-isotropic ($[0°/\pm45°/90°]_s$) laminates.
3. It can capture the behavior of a range of CMCs, including those with stiff or compliant matrices.

The outline of the article is as follows. First, the results of mechanical tests (uniaxial tension at 0°, tension at 45°, and Iosipescu shear) for a commercial SiC/SiCN woven CMC are presented. Second, these data are combined with previously-reported results for other material systems and used to assess two existing phenomenological models for CMC laminates: notably, that of Genin and Hutchinson [2] and an adaptation of the model of Hahn [20]. The two models are distinguished by their respective predictions of the relationship between the mechanical responses in shear and in 45° tension. Notably, the shear/tensile cracking stress ratios from the GH model and the Hahn model are 1 and 1/2, respectively. The actual behavior, from both the experimental data and from theoretical considerations, is roughly bounded by these two extremes. Third, a new phenomenological elastic/plastic model is proposed. It is based on the deformation theory of plasticity and it combines features of both the GH and Hahn-type models. Importantly, it allows for arbitrary values of the shear/tensile cracking stress ratio. The predictive capabilities of the model are assessed in two ways: (i) by comparisons with measured inelastic responses in 45° tensile stress–strain curves, and (ii) comparisons of computed and measured displacement and strain fields in open-hole tension tests. The errors in the predicted open-hole tensile results are computed and rationalized in terms of the degree of non-proportional stressing that occurs during inelastic straining in the test geometry of interest.

2. Materials and experiments

2.1. Material

The material investigated in the present study comprises 8 plies of Hi-Nicalon (SiC) fibers in an 8-harness satin weave, a BN interface coating, and a SiCN matrix made through a combination of slurry infiltration and precursor impregnation and pyrolysis (S-200H, COI Ceramics, Inc., San Diego, CA). The finished composite panel was 2.25 mm thick. Optical micrographs (Fig. 1) reveal some remnant porosity and extensive microcracking in the matrix. These defects play a significant role in the average matrix modulus and strength, as manifested in the mechanical test results presented below.

2.2. Experimental procedures

Three sets of mechanical tests were performed: uniaxial tension at 0° and at 45°, and shear parallel to the two fiber directions. Either two or three specimens were used for each set. The test specimens were designed to accommodate the limited quantity of available material.

For both sets of uniaxial tension tests, a dog-bone geometry was employed. For the test at 0°, the gauge width was 12.4 mm and the gauge length was 25.4 mm; for the test at 45°, the gauge width was 7 mm and the gauge length was 14 mm. Fiberglass tabs were adhered using a commercial epoxy to the ends of the tensile specimens to promote even load transfer. The specimens were loaded using hydraulic wedge grips.

Shear properties were measured using the Iosipescu test. Specimen design was broadly in accordance with the pertinent ASTM standard (ASTM D5379), with one notable exception: the V-notch angle was selected to be 105°, instead of 90° (Fig. 2). A notch angle larger than 90° is desirable for testing orthotropic materials, since it produces somewhat smaller transverse stresses in the central ligament [24]. The specimen was loaded using a standard Iosipescu test fixture (Wyoming Test Fixtures, Inc.).

All specimens were instrumented using strain gauges on one surface and 3-D DIC (VIC-3D, Correlated Solutions, Inc.) on the other. For the Iosipescu shear test, a [0°/90°] stacked strain gauge rosette (Vishay Micro Measurements, CEA-13-062WT-120), oriented at ±45° to the specimen axes, was used; a single (axial) strain gauge was used for the tension tests. To enable use of digital image correlation, an artificial speckle pattern was created on the sample surface. This was accomplished by first painting the specimen surface uniformly white and then spraying fine black speckles onto the surface using an airbrush. The speckle size, measured using an autocorrelation technique, was approximately 50 μm. Two digital cameras (Point Grey Research Grasshopper), each with

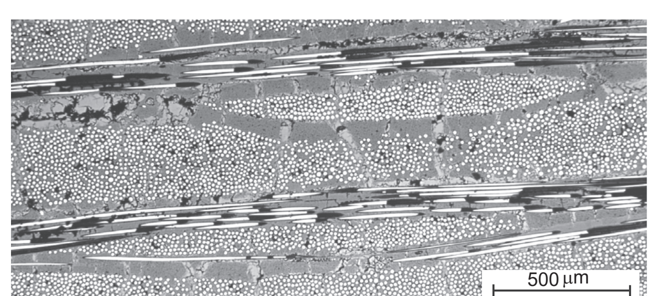

Fig. 1. Optical micrograph of a polished cross-section through the composite.