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Design of filament–wound domes based on continuum theory and non-geodesic roving trajectories

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ABSTRACT

In this paper the optimal shapes and fiber architectures of non-geodesics-based domes for pressure vessels are determined upon the condition of equal shell strains. Based on the continuum theory and the non-geodesic law, the system of differential equations governing the optimal meridian profiles is derived. A specific function is chosen to describe the slippage coefficient distribution for the desired non-geodesic path, in order to ensure C^1 continuity of the roving paths when passing the dome–cylinder conjunction. Next, the meridian profiles are determined for various material anisotropies; the related winding angle developments of non-geodesic trajectories are also presented. The performance factors of non-geodesics-based optimal domes are obtained using various slippage coefficients and polar opening radii. The results show that the structural efficiency of the dome improves with increasing slippage coefficient. It is concluded that the non-geodesics-based dome designed using the present method gains better performance than the one relying on geodesics.

1. Introduction

Filament–wound composite pressure vessels have gained widespread application in the field of aerospace, transportation, chemical and underwater engineering. The quality of composite pressure vessels is usually expressed in terms of the performance factor, which is defined as pressure times volume divided by weight (PV/W). From this formulation it becomes evident that accurate estimations of the burst pressure and the resulting vessel weight are of vital importance. For the calculations of the required properties one can distinguish two approaches: the netting theory and the continuum theory. In both approaches, the basic idea is to maximally utilize the available strength. During pressurization, the structure is under uniform through-thickness strain and consequently no bending or discontinuity stresses are here assumed. The netting theory provides results according to analytical or graphical approaches [1–3]; however, it has a major defect in that design calculations are solely based on fiber strength and the matrix effect is not considered. The mechanical and structural performance is predicted by neglecting the contribution of the resin system. In addition, the continuum theory is more accurate and shows the ability to cover the complete range from fully orthotropic to entirely isotropic materials. Considering the current availability of computational resources and the accessibility of the numerical operations that have to be undertaken, the continuum theory is actually preferred, unless the designer seeks for a preliminary dimensioning procedure. One should note that the netting approach is a special case of the continuum theory.

The geometrical determination of the dome is the major part of designing pressure vessels. Various methods have been presented for determining optimal domes, based on the continuum theory. De Jong [4] compared the shapes of optimal profiles determined by the netting and the continuum theory and indicated that the geometry and performance of optimal domes are dependent on the elastic properties of the materials used. Hojjati et al. [5] evaluated the effect of mechanical properties of composites on the dome profiles and proved that the matrix properties have a major role in the dome design. Vasiliev et al. [6,7] derived the optimality conditions for a pressure vessel based on the classical lamination theory and outlined the shapes of optimal dome profiles corresponding to various anisotropic characteristics. Liang et al. [8] presented the optimal design of dome contours by maximizing the shape factor and evaluated the effect of the dome depth on the structural performance. Zu et al. [9] developed an optimal design method for the class of articulated pressure vessels comprising various dome cells that are axially stacked on each other. Fukunaga and Uemura [10] determined optimal meridian shapes using several failure criteria and presented an analytic approach for the optimal design of dome structures. Tackett et al. [11] conducted a combined analytical and experimental effort to characterize dome reinforcement requirements for intermediate modulus carbon/epoxy pressure vessels and evaluated the influence of shallow dome profiles on their performance. De Vita et al. [12] outlined the process simula-

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tion in filament winding of composite structures. Blachut [13,14] investigated the optimal meridian shape and thickness distributions in a filament wound dome closure, and discussed the relevant details of manufacture, testing and numerical analysis of the torispherical heads.

Although the continuum-based dome design is sufficiently covered in the literature, there are some deficiencies. The majority of previous research has merely considered the dome design based on geodesic trajectories, and overlooked the application of non-geodesics to the optimal design of meridian profiles and their related roving paths. Geodesics represent the shortest paths connecting two arbitrary points on a continuous surface, and they show great stability on a curved surface and calculability with the Clariaut equation [15]. However, as geodesic paths are entirely determined by the underlying meridian profile and initial winding angle, their geometry, combined with the requirement for tangential placement of the rovings at the polar areas, certainly limits the design opportunities of domes [16,17]. A typical example of this restriction is the limit for improving the structural performance of domes. The possibility appears now for applying friction-based non-geodesics to the design of roving trajectories for pressure vessels.

An optimal dome design relies on the most efficient distribution of laminate thickness and stress, in order to maximize the structural performance. As the strength-dominated and manufacturing-dominated thickness distributions do generally not match, the laminate strength cannot be maximally utilized. A well-known solution for this problem comprises geodesic-isotensoid designs, based on the netting analysis. However, geodesics do generally not result in optimal solutions for vessel design problems whereby the matrix strength has to be taken into account [7,9,18]. It is thus desirable to exploit non-geodesics to enlarge the design space for obtaining the optimal meridian shapes and related roving trajectories, so that the minimum required thickness distributions as determined by strength analysis can maximally coincide to the manufactured thickness distributions as determined by the winding process. In Section 2, we present the differential equations for determining non-geodesics on the surface of a dome. Then, the optimality condition of equal shell strains for a pressure vessel is derived (Section 3), based on the minimum strain energy criterion, in order to maximize the structural stiffness and load bearing capacity. In Section 4 the non-geodesics-based optimal meridian profiles are determined with the aid of the equal-strains condition, and the influence of the orthotropy on geometrical issues, such as the resulting meridian shape and the tangentiality of the rovings at the polar opening, is evaluated. The method is then demonstrated by three typical composite materials, reflecting on the most general design cases of domes (Section 5). To assess the effect of non-geodesic paths on the structural performance of the dome, we calculate and compare the performance factors of non-geodesics-based optimal domes for various slippage coefficients and polar opening radii. The shell thicknesses are determined by the combination of a strength criterion and the geometric (winding) condition. Lastly, the distributions of laminate stresses are obtained in order to illustrate that non-geodesics-based optimal domes are relatively thinner than the geodesics-based ones, mainly triggered by the efficient utilization of the laminate strength.

2. Non-geodesic trajectories

The vector representation of a dome structure in polar coordinates is:

$$S(\theta, z) = \{r(z)\cos\theta, r(z)\sin\theta, z\} \tag{1}$$

where r, z denote the radial and the axial distances, and θ stands for the angular coordinate in the parallel direction, as shown in Fig. 1.

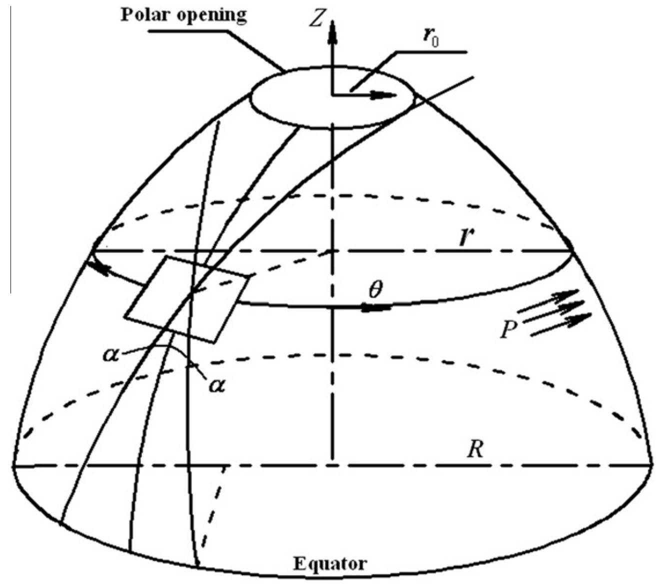

Fig. 1. A generic shell of revolution.

The slippage coefficient λ represents the slippage tendency of the roving bundles that are placed on the supporting surface, and is defined as the ratio of the geodesic curvature, k_g, to the normal curvature, k_n [19]:

$$\lambda = k_g/k_n \tag{2}$$

The first fundamental form of $\mathbf{S}(\theta, z)$ is [15]:

$$E = r^2, \quad F = 0, \quad G = 1 + r'^2 \tag{3}$$

where r' is the first derivative of r with respect to z. Substitution of Eq. (3) into the *Liouville* formula [15] leads to the expression for the geodesic curvature k_g:

$$k_g = \frac{d\alpha}{dl} + \frac{r' \sin\alpha}{r\sqrt{1 + r'^2}} \tag{4}$$

where α is the angle between the roving path and the meridional direction of the dome, i.e., the winding angle, as shown in Fig. 1; l is the arc length along the roving path.

The normal curvature, k_n, can be expressed in terms of the main curvatures in the meridional and parallel directions. According to the *Euler* formula [15], we obtain:

$$k_n = -\frac{r''}{(1 + r'^2)^{3/2}}\cos^2\alpha + \frac{1}{r\sqrt{1 + r'^2}}\sin^2\alpha \tag{5}$$

where r'' is the second derivative of r with respect to z. By substituting Eqs. (4) and (5) into (2), the differential equation for the non-geodesic paths becomes:

$$\frac{d\alpha}{dl} = -\lambda\left(\frac{r''}{(1 + r'^2)^{3/2}}\cos^2\alpha - \frac{1}{r\sqrt{1 + r'^2}}\sin^2\alpha\right) - \frac{r' \sin\alpha}{r\sqrt{1 + r'^2}} \tag{6}$$

Since the fiber has an orientation α with respect to the meridian (Fig. 2), the relation between dz/dl and the winding angle α can be derived as follows:

$$\frac{dz}{dl} = \frac{dz}{ds_{meridian}} \cdot \frac{ds_{meridian}}{dl} = \frac{dz}{\sqrt{1 + r'^2}dz} \cdot \cos\alpha = \frac{\cos\alpha}{\sqrt{1 + r'^2}} \tag{7}$$

Substitution of Eq. (7) into (6) leads, after some arrangements, to the expression for determining non-geodesic trajectories with respect to α and z: