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A novel method for measuring the through-thickness shear moduli of anisotropic plates from surface deformation measurements

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ABSTRACT

The present study is dedicated to a novel method aimed at measuring the through-thickness shear moduli of anisotropic plates from surface deformation measurements. The first part of the paper presents the derivation of a relationship linking the through-thickness average of the interlaminar shear strain to surface deformations. A short description on how this feature could be made use of through the Virtual fields Method is then given. The second part of the paper describes the validation of this new relationship on simulated finite element data, and then, on experimental data obtained through speckle interferometry. It is shown that the practical use of this technique is very challenging but initial results are promising, provided that some correction is applied to the raw data.

1. Introduction

The measurement of the through-thickness moduli of composite plates is an important issue, as more and more thick composites are used in primary structures in the aerospace industry for instance. In particular, the transverse or interlaminar or through-thickness shear moduli are very important to predict the stiffness of moderately thick or curved panels. This is enhanced by potentially high anisotropy ratios in composites (ratio between Young's modulus and the corresponding interlaminar shear modulus).

A number of techniques have been reported over the years to provide these interlaminar shear moduli. Unfortunately, as stated in a recent paper [1]:

> *"A noticeable void in current literature is the lack of a test method for determining the interlaminar shear modulus. This is largely attributable to the fact that conventional methods of direct stress and strain measurements cannot be easily adapted for the measurement of interlaminar properties."*

Moreover, as also stated in the same paper:

> *"For example, utilising these conventional methods for determining the interlaminar shear modulus requires extremely thick composite coupons to be manufactured, which has proven to be very difficult and costly."*

The present author is in complete agreement with these two statements. Only a few testing techniques exist for the determination of the through-thickness shear moduli of plates. Several sources of interlaminar shear test method comparison and assessment can be found in the literature, for instance [2–4]. The objective here is not to provide a thorough review of such test methods but to browse through the most popular techniques to underline the novelty of the proposed approach.

A first well-known method is the three-point bending test on short span beams [2]. This test is usually employed to give information on the interlaminar shear strength. There has been some attempts at using it for the interlaminar shear modulus [5] but there is the issue of indentation at the loading points. This test is usually considered not suitable for modulus measurement [2,3]. Another rather underexplored possibility is the torsion test on rectangular bars [6–9]. Ideally, it requires the use of strain gauges bonded on both the in-plane and through-thickness sides to enable the identification of both the in-plane and interlaminar shear moduli. However, this still requires rather thick specimens and the data processing relies on an analytical solution of the mechanical problem, hence a strong restriction in shape and boundary conditions. Certainly one of the most popular methods for interlaminar shear modulus identification is the Iosipescu test (or double V-notch test, ASTM standard D5379). Several studies have exhibited convincing results on the measurement of the interlaminar shear modulus of a wide range of composites, for instance [3,10–14]. However, the main disadvantage is that rather thick panels have to be manufactured (about 20 mm for the standard fixture), even though this requirement may be slightly relieved if tabs are used such as in [14]. Other less popular test methods have been used, such as the cube test [15] or the inclined double notch specimen [16], for instance. Some other techniques have also been reported in [17], this list being non-exhaustive. It must also be noted that inverse

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procedures have been attempted to relax the requirements in test geometry and loading [1]. Basically, all these methods (except the short beam test) require to perform some kinematic measurements on the interlaminar plane. For thin plates (like 1 or 2 mm thick), this may be very unpractical or altogether impossible for specimens like tubes. Moreover, all these usual test methods (except that in [1]) rely on strict specimen shape and loading arrangement for either an analytical or an approximate solution of the associated mechanical problem.

An alternative strategy consists in performing vibration tests on plates. Initially developed for the identification of the in-plane stiffness components, these strategies, based on model updating (often finite element model updating), have been extended to the interlaminar shear moduli using higher order shear plate theories [18–20]. The results are satisfactory provided that the plates are thick enough for their eigenfrequencies or mode shapes to be sensitive to interlaminar shear. Nevertheless, the results are usually difficult to compare to mechanical quasi-static tests because of the effect of strain rate and strain levels, vibration tests giving rise to only small strains in the plates.

The objective of this paper is to present an innovative alternative to measure the through-thickness shear moduli of composite plates from bending tests and surface full-field measurements of the three displacement components, using the virtual fields method. First, the general theoretical framework is given and then, both numerical and experimental validations are performed on a simple cantilever-like test on a slender unidirectional carbon/epoxy composite specimen.

2. Theory

Let us consider a plate of in-plane dimensions L and l and thickness h, submitted to a set of bending loads. As an example, Fig. 1 shows such a plate submitted to a bending test already used in another study [21]. The material is supposed orthotropic in the test plate coordinate system (x,y,z). Identifying the four bending stiffness components D_{xx}, D_{yy}, D_{xy} and D_{ss} is relatively easy, in particular when full-field measurements are available (see [21–23], for instance) but obtaining the two through-thickness shear moduli G_{xz} and G_{yz} as well is much more challenging. This section presents a procedure to do this by including the in-plane displacements in the analysis.

2.1. Higher order shear plate theory

In situations where the effect of through-thickness shear deformation cannot be neglected with respect to the in-plane strains, a general formulation of the displacement field can be given as [24]:

$$\begin{cases} u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + f(z)\gamma_x^0(x,y) \\ v(x,y,z) = -z\frac{\partial w(x,y)}{\partial y} + f(z)\gamma_y^0(x,y) \\ w(x,y,z) = w(x,y) \end{cases} \quad (1)$$

where u, v and w are the three components of the displacement field. Computing the strains from this equation, one has:

$$\begin{cases} \varepsilon_{xx}(x,y,z) = -z\frac{\partial^2 w(x,y)}{\partial x^2} + f(z)\frac{\partial \gamma_x^0(x,y)}{\partial x} \\ \varepsilon_{yy}(x,y,z) = -z\frac{\partial^2 w(x,y)}{\partial y^2} + f(z)\frac{\partial \gamma_x^0(x,y)}{\partial y} \\ \varepsilon_{zz}(x,y,z) = 0 \\ 2\varepsilon_{xy}(x,y,z) = -2z\frac{\partial^2 w(x,y)}{\partial x \partial y} + 2f(z)\left(\frac{\partial \gamma_x^0(x,y)}{\partial y} + \frac{\partial \gamma_y^0(x,y)}{\partial x}\right) \\ 2\varepsilon_{xz}(x,y,z) = f'(z)\gamma_x^0(x,y) \\ 2\varepsilon_{yz}(x,y,z) = f'(z)\gamma_y^0(x,y) \end{cases} \quad (2)$$

It clearly appears from this formulation that $f(z)$ drives the warping of the section and, through its derivative, the through-thickness distribution of shear. It must be pointed out that complying to the top and bottom free surface boundary conditions, one must have $f'(h/2) = f'(-h/2) = 0$ and that since the in-plane displacements are zero on the mid-plane, $f(0) = 0$. Finally, f must be an odd function to ensure that $u(x,y,h/2) = -u(x,y,-h/2)$ and $v(x,y,h/2) = -v(x,y,-h/2)$ and at least C^1. From this general situation, several popular formulations have been proposed:

- $f(z) = 0$: this leads to the Love-Kirchhoff theory of thin plates where the through-thickness shear strains are neglected with respect to the in-plane strains. It can also be translated as the fact that plane normal sections remain plane and normal to the deformed mid-plane.
- $f(z) = z$: this leads to what is usually referred to as the Reissner–Mindlin theory. In this case, since $f'(z) = 1$, the through-thickness shear strains are assumed constant through the thickness, which clearly violates the free surface boundary conditions and therefore, requires shear correction factors.

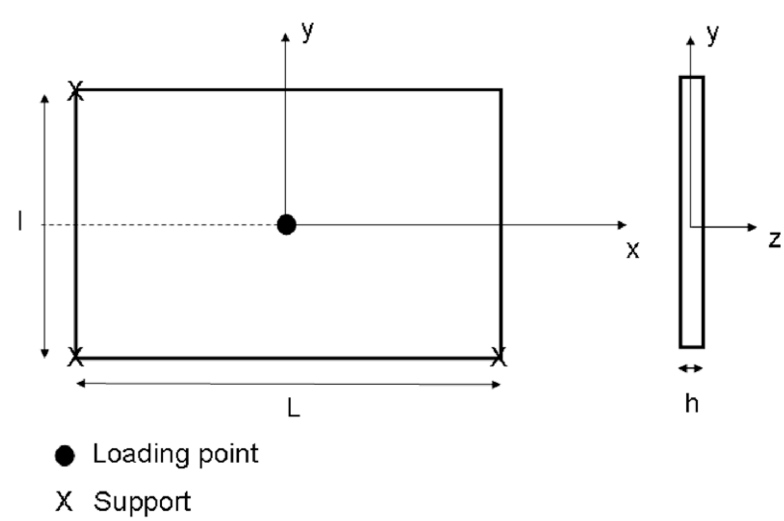

Fig. 1. Example of test configuration.