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Asymptotic model for deep bed filtration

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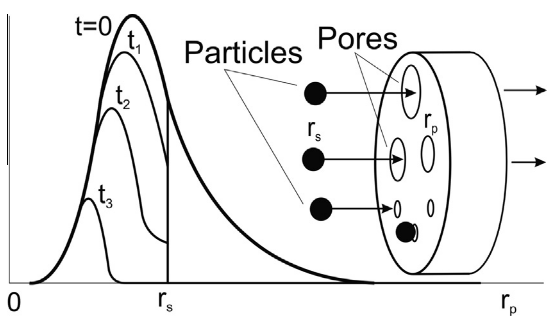
HIGHLIGHTS

- Analytical asymptotic model for non-linear deep bed filtration.
- Explicit formulae to treat long-term laboratory data on colloidal suspension flow.
- Data treatment from the period 5–10 times longer than that used by classical method.
- Fewer laboratory tests required to adjust micro scale population balance model.
- Accuracy–uncertainty criteria of colloidal flow laboratory and asymptotic modelling.

GRAPHICAL ABSTRACT

Evolution of pore size distribution for size exclusion deep bed filtration in porous media.



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ABSTRACT

Asymptotic solution for non-linear stage of colloidal suspension flow in porous media is developed. The expansion is performed behind the concentration front; the zero order approximation coincides with the short-time solution of the linearised system. Using the first order approximation allows significantly enhancing the validity time period for the analytical model if compared with the linearised solution, allowing using the long-term experimental breakthrough concentration history for the model adjustment. Laboratory injection tests for three size colloids are performed. The asymptotic solution is used to tune the model parameters from the breakthrough histories of two size particles; good quality of matching is observed. The breakthrough concentration history for the third size particles is compared with the prediction by the adjusted model; good quality prediction is observed. The above serves for validation of the asymptotic method for the model tuning and prediction of non-linear colloidal suspension flow in porous media.

1. Introduction

Colloidal suspension flow in porous media with particle capture is essential for numerous chemical, environmental and petroleum technologies. It takes place in industrial filtering, size exclusion chromatography, water production by artesian wells, industrial waste disposal, aquifer remediation, contamination of aquifers by viruses and bacteria, low quality water injection in oilfields, and fines migration in low consolidate and high-clay-content oil reservoirs [1–11].

Planning and design of the above chemical engineering and petroleum technologies are based on mathematical modelling.

Deep bed filtration of colloidal suspensions in porous media is described by the equation of mass balance of suspended and retained particles and the retention rate equation [12–14]:

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Nomenclature

a	reciprocal of the front speed
C	suspended particle concentration distribution by sizes, L^{-4}
C_v	variance coefficient
c	total suspended particle concentration, L^{-3}
D	accuracy of the asymptotic model
E	normalised least square deviation of the model from the measured data
f_a	accessible fractional flow function
H	pore concentration distribution by sizes, L^{-3}
h	total pore concentration (density), L^{-2}
j	jamming ratio
k	permeability, L^2
k_1	conductance in a single pore, L^4
l	inter-chamber distance, L
p_a	particle attachment probability
r_p	radius of a pore, L
r_s	radius of a particle, L
S	dimensionless captured particle concentration
t	time, T or dimensionless
t_f	total injection time
U	Darcy velocity, LT^{-1}
u	combined standard uncertainties
x	linear coordinate, L or dimensionless
y	independent variable in the differential-integral system

Greek letters

δ	Dirac delta function
δ_C	relative combined standard uncertainties for suspended concentration
δ_S	relative combined standard uncertainties for retained concentration
ζ_b	zeta potential for glass beads, $ML^2\,T^{-3}\,I^{-1}$
Λ	dimensionless filtration coefficient
λ	filtration coefficient, L^{-1}
ν	flux reduction factor in a single pore
σ	total concentration of captured particles, L^{-3}
ϕ	porosity
ϕ_a	accessible porosity
Σ	captured particle concentration distribution by sizes, L^{-4}
ψ_s	surface potential for particles, $ML^2\,T^{-3}\,I^{-1}$

Subscripts

a	accessible
C	suspended concentration
S	retained concentration
p	pore
0	initial value
0,1,2	the zero, first and second order of asymptotic solution

Superscripts

0	boundary value at the inlet
L	boundary value at the outlet

$$\frac{\partial}{\partial t}(\phi s c + \sigma) + U\frac{\partial}{\partial x}(c f_a) = 0,\ f_a = 1 \tag{1}$$

$$\frac{\partial \sigma}{\partial t} = \lambda(\sigma) U c \tag{2}$$

where $\lambda(\sigma)$ is the filtration coefficient and s is the accessibility factor that is equal to the ratio between the porosity accessible to particles and the overall rock porosity. The fractional flow function f_a corresponds to the fraction of the overall flux moving via the accessible pores [15,16].

At low retained concentration σ, the filtration coefficient becomes constant $\lambda(0)$ and the system becomes linear, allowing for exact solution for clean bed injection with constant concentration c^0:

$$c(x,t) = \begin{cases} \exp\left(-\frac{\lambda}{f_a}x\right), & t > a \\ 0, & t < ax \end{cases},\ \sigma(x,t) = \begin{cases} \lambda(t-ax)\exp\left(-\frac{\lambda}{f_a}x\right), & t > ax \\ 0, & t < ax \end{cases},\ a = \frac{\phi s}{f_a} \tag{3}$$

More complex solutions for deep bed filtration accounting for diffusion are presented by van Genuchten and Alves [17], Ziskind et al. [18] and Guerrero et al. [19,20].

Population balance models for colloidal suspension flows in porous media include particle size distributions of suspended and retained concentrations and pore throat size distribution [21–24]. The models describe the distribution changes due to different attachment and straining rates of different size particles in different size pores.

Other micro scale models include stochastic trajectory analysis [25,26], random walk master equation [27–32] and direct pore scale simulation [14,33].

The model adjustment from the laboratory data is necessary for reliable process prediction by the mathematical modelling. The adjustment of deep bed filtration models is performed by solving the inverse problems [34,35] or by least square tuning of the model parameters [36]. Presently the tuning is performed using the analytical solution for the linearised population balance model (3), where the breakthrough concentration values $c(1,t_{br})$ for particles with different radii r_s are used to determine the filtration coefficient $\lambda(r_s)$ [37]. So, only one tuning constant is retrieved per one size of the injected particles, i.e. one measurement (3) provides with just a single constant for tuning of the population balance model.

It significantly limits the experimental information necessary for the population balance model adjustment due to limited number of particle sizes available on the market. The amount of experimental information can be significantly increased by using the suspended concentration histories long after the breakthrough moment, but the expressions (3) are valid during short after-the-breakthrough periods.

Moreover, the accurate determination of suspension concentration in the breakthrough time cannot be done from one measurement, so the average from the "neighbouring" points is taken. Yet, the retention concentration grows after the breakthrough moment; it is not accounted for in the solution (3) and decreases the accuracy of $c(L,L/U)$ calculation. Despite the method (3) is widely spread, the time period after the breakthrough where the method is valid has never been estimated.

In the present work, the asymptotic model for non-linear deep bed filtration processes is developed. The zero order approximation coincides with the known solution of the linearised system (3). The first order approximation allows prolonging significantly the time period, where the breakthrough concentration can be