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Dynamic crack propagation along the interface with non-local interactions

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ABSTRACT

A model for crack propagation along a non-locally pre-stressed interface in ceramic materials is studied. The problem is analysed on a discrete chain of oscillators, with local and non-local interactions. The pre-stressed state is defined by the stiffnesses of the non-local links. Small negative stiffnesses of such links reveals the existence of crack speeds supporting stable crack propagation, which is not possible in a structure which has only local interactions.

1. Introduction

Ceramic materials possess certain superior properties, which make them attractive for use in fields including structural engineering, bioengineering, and aeronautics. However, these materials have some weaknesses, especially brittleness, as a result of low fracture toughness. These disadvantages may be overcome by several methods, including transformational toughening, fibrous toughening [28]. The authors of [13] used FEM to study crack toughening mechanisms (parallel arrays of cracks and crack kinking due to plastic dissipation) for layered and particulate solids, which may be applicable to ceramic composites. Apart from being used as monomaterials, ceramics have been extensively used in complex structures and composites, where differing ceramic materials are joined together or with metals [22]. Notably, the superior properties of ceramic-metal joints see them widely used in various structures and situations where monoceramic materials are unsuitable. Also, since the fabrication of large ceramic structures is a complex and expensive process, it can be advantageous to use ceramics only for the parts which require their use.

There are several techniques used in joining ceramic to metal, including various brazing methods [25] and pulsed laser deposition [12]. Even though it is possible to produce a relatively strong ceramic/metal interface using several techniques, residual stress concentrated near the ceramic-metal joints may significantly decrease the life time and workability of the structure. The distribution of the residual stress presented in [14,20,21] reveals clear evidence of a stress concentration close to the ceramic/metal interface. This, in turn, may lead to microcracks in these regions, and their further propagation as a macro-crack along the interface.

In the present paper, we analyse the properties of such interfaces at the micro-level, where we use for this purpose a one-dimensional discrete model, and consider stead-state crack propagation along the interface. This analysis is based in considering a chain of oscillators attached to a rigid surface with linear springs. The crack is modeled as a lack of such support for half of the chain. The interactions between the various oscillators are modeled by local linear springs and next-nearest-neighbour interactions. Similar models featuring non-local interactions were used to study phase transitions [23,24], and to perform numerical simulations of crack propagation in structured media [3].

In our analysis, we assume that the non-local interactions, resulting from varying concentrations of different atoms along the joint interface [12,14,20,21], are small in comparison with the local interactions. In other words, in the model under consideration, we study the effect of small values of the spring constant in comparison with the local stiffness. The recent paper [5] demonstrates the effects that arise from non-local interactions. The authors show that the anisotropy of the elastic structure may lead to some unexpected effects, where the crack may propagate at a lower speed in comparison with similar discrete structures exhibiting only local interactions.

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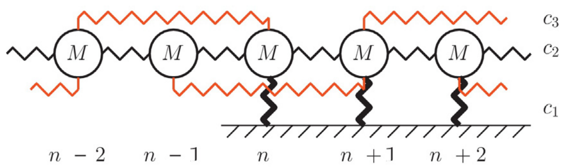

Fig. 1. Chain of oscillators with equal mass M, connected to a substrate with springs of stiffness c_1 (fat lines), closest neighbours with springs of stiffness c_2 (normal lines), and second closest neighbours with springs of stiffness c_3 (red lines), where n^* represents the crack tip. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In this paper, we are interested in a different effect. Specifically, we assume small variations in the interface properties can exist in combination with a constant average stiffness of the interface. In detail, we consider small perturbations of the non-local stiffness, taking both positive and negative values. The negative values of stiffness are referred as 'elements with negative stiffness', where there are several papers on this topic, e.g. [26,27,6]. These 'negative stiffness elements' may be used to model mechanical systems with stored energy, and the above mentioned papers reveal that these elements may effect the damping properties of the mechanical system.

The solution of this problem is based on the ideas proposed by Slepyan [19], which have been further developed in a number of publications, for example [8–11,15–18], with reference to the local interactions occurring in various lattice structures. The technical solution of the problem requires specific mathematical techniques, and is performed in [5]. In the present paper we focus only on the consequences of the non-local interactions with negative stiffness, demonstrating that such configurations may lead to a considerable difference in the strength properties of the interface.

2. Problem formulation

Let us consider a discrete model with nonlocal interactions, represented by a chain of masses, and analyze the crack propagation in the structure (Fig. 1). The following notations are introduced: M is the mass of a particle, c_1 is the spring constant of the links between the particles and a substrate, c_2 is the stiffness of the bonds between the closest neighbors, and c_3 is the spring constant between the second closest neighboring particles. We suppose that the particles are evenly distributed, with a separation of length a, and that the vertical springs also have the same equilibrium length. The coordinate $n^*(t)a = vt$ represents the location of the crack tip, which propagates with a constant speed v from left to right. We study the effect of introduced non-local interactions, i.e. the magnitude of c_3 in comparison with c_1 and c_2.

The equations of motion for the mechanical system considered take the form

$$M\frac{d^2u_n}{dt^2} = c_3(u_{n+2}+u_{n-2}-2u_n)+c_2(u_{n+1}+u_{n-1}-2u_n)-c_1u_n, \quad n \geq n^*,$$
$$M\frac{d^2u_n}{dt^2} = c_3(u_{n+2}+u_{n-2}-2u_n)+c_2(u_{n+1}+u_{n-1}-2u_n), \quad n < n^*, \tag{1}$$

where $u_n(t)$ is the horizontal displacement of the nth oscillator from the equilibrium position. The following fracture criterion is used:

$$\begin{cases} u_{n^*} = u_c, \\ u_n < u_c, \quad n > n^*, \end{cases} \tag{2}$$

where u_c is a critical value of the displacement field at the crack tip. The first equality allows us to find a threshold loading condition, which is determined in further analysis. We note that a natural choice for the criterion would be a critical magnitude of the absolute value of displacement of the oscillator $|u_{n^*}|$ at the crack tip. Such a condition, however, is not always consistent with the assumption of steady-state crack propagation and we refer the prospective reader to the paper [11], where both criteria are considered and discussed.

We assume throughout the paper that the crack propagates from the left to the right with a constant speed v,

$$v < v_c, \quad v_c^2 = \frac{c_2+4c_3}{M}a^2, \tag{3}$$

where v_c is the sound speed in the broken part of the chain ($n < n^*$). Since we are studying the effects of the material parameters, all computations have been performed using the dimensionless values of M, c_j. Let us fix the value of $c_2 + 4c_3$ for all the computations:

$$c_2 + 4c_3 = 1. \tag{4}$$

This linear combination of c_2 and c_3 represents the effective stiffness, which tends to some macroscopic stiffness in the limit $a \to 0$. For all further examples, we let $M = 1$. In order to track the relevance of low levels non-local interaction we set:

$$|c_3| < \frac{1}{12}. \tag{5}$$

The problem considered corresponds to a mode III fracture. The authors of [8] solved a similar problem, within a different physical context that is suitable for the analysis of mode II fracture.

3. Energy release rate and displacement profile ahead of the crack front

The details for the full solution can be found in [5]. Here, we proceed directly to the numerical results, and will concentrate solely on the computation of the most decisive parameter in fracture mechanics, the Energy Release Rate (ERR).

We describe the first equation from (2) as the deformation fracture criterion for damage to the nearest spring to the crack tip. Following [16], we numerically evaluate the dependence of the energy ratio G_0/G on the associated crack speed. We here use G to denote the global energy release rate enforcing the crack propagation, while G_0 is the local energy release rate determined by the critical value of u_c and the stiffness of the broken link c_1. We see that the ratio G_0/G is independent of the value u_c [16]. In the discrete structure dynamic, crack propagation is qualitatively different to the propagation seen in continuous media. Indeed, as was shown in [19], the global ERR, G, can be expressed as a sum of the fracture energy G_0 and the energy carried by the elastic waves that radiate from the crack tip. Consequently, as the ratio G_0/G decreases, more energy is released from the crack tip by those elastic waves in the course of fracturing. Thus, only part of the global energy is consumed by the fracture itself ($G_0 < G$).

The plots of the energy release ratio G_0/G are shown in Fig. 2, for different values of the parameters c_1, c_2, c_3. In the numerical simulation we used $c_1 = 0.5$, 1, 2 and $c_3 = 1/16$, 0, $-1/32$, $-1/16$, while the value of c_2 was determined according to Eq. (4). In the case $c_1 = 1$, the displacement profile calculations were performed for two choices of crack speed, $v = 0.5v_c$ and $v = 0.2v_c$, in Fig. 3(a and b), respectively, while Fig. 3(c) provides an enlarged view of the profiles near the crack tip in the case of $v = 0.2v_c$. The plots show the dependence $u(\eta)/u_c$, where

$$\eta = n - vt. \tag{6}$$