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A finite element framework for the interplay between delamination and buckling of rubber-like bi-material systems and stretchable electronics

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ABSTRACT

In this study, a finite element (FE) framework for the analysis of the interplay between buckling and delamination of thin layers bonded to soft substrates is proposed. The current framework incorporates the following modeling features: (i) geometrically nonlinear solid shell elements, (ii) geometrically nonlinear cohesive interface elements, and (iii) hyperelastic material constitutive response for the bodies that compose the system. A fully implicit Newton–Raphson solution strategy is adopted to deal with the complex simultaneous presence of geometrical and material nonlinearities through the derivation of the consistent FE formulation. Applications to a rubber-like bi-material system under finite bending and to patterned stiff islands resting on soft substrate for stretchable solar cells subjected to tensile loading are proposed. The results obtained are in good agreement with benchmark results available in the literature, confirming the accuracy and the capabilities of the proposed numerical method for the analysis of complex three-dimensional fracture mechanics problems under finite deformations.

1. Introduction

Multi-layered systems integrating different materials are present in many engineering applications and natural systems. One of the most common configuration consists of coating a flexible thick substrate by a much stiffer thin layer. These coating layers usually incorporate a functional device, namely biomedical sensors or microchips, among others, and at the same time serve to protect the substrate from the environmental conditions as is the case of ceramic materials which act as thermal barriers [1–7]. A potential area of use of bi-layered systems is flexible-electronics, which comprises very promising applications such as printable solar cells, flexible tilt sensors, among many others [8]. In particular, flexible-electronic devices are generally realized with a pattern of silicon-based stiff islands bonded over polymeric substrates. Due to the mismatch between the mechanical properties of the layers and external loadings acting on the device, debonding of the islands may occur both during manufacturing and service conditions [2].

The analysis of the mechanical behavior of these systems has attracted an intensive research activity in the last few years. Numerous studies have thoroughly investigated the different modes of instabilities of layer-substrate systems subjected to uni-axial compressive loadings, which range from creasing, folding, wrinkling, buckling-delamination, and crumpling, among others. These complex phenomena have been also observed in experiments, with regard to polymers [9–12] or hard coatings [4], among many others.

The prediction of critical loads leading to these instabilities can be achieved via semi-analytical methods as in [13–18]. Attempts to investigate the concomitance of wrinkling and delamination has also been addressed in [14,19], where semi-analytical formulations have been invoked to predict the critical loading for the onset of delamination and to simulate the post-critical branch.

Recently, a multi-scale finite element framework incorporating the asymptotic numerical method has been proposed in [20] to follow the post-buckling evolution path and predict secondary bifurcations in stiff thin layer systems perfectly attached on a soft substrate.

Other studies in this area have been devoted to the analysis of the effect of the layer growth/swelling and thermal strain, see [21] and the references therein given. Recently, Lagrance et al. [22] have

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extended this line of research to the analysis of cylindrical layer-substrate systems subjected to pressure loading, deriving a closed form solution that is used to conduct the corresponding stability analysis.

The stability analysis of multi-layered systems has not been exclusively limited to uniaxial compressive conditions. Several investigations have reported valuable results with regard to the development of structural instabilities under finite bending conditions [11,23] since these situations are commonly observed in a wide range of engineering applications. In this regard, Roccabianca et al. [24] characterized the bifurcation phenomena in elastic blocks experimentally observed by Rivlin [38] using the so-called *compound matrix method*, providing an important explanation to the occurrence of long-wavelength bifurcation modes in the post-critical branch.

The present study investigates delamination of thin stiff layers from soft stretchable substrates using the three-dimensional finite element framework recently proposed in [7], which combines finite deformation solid shell [25] and cohesive interface elements [26]. The analysis here presented further extends this framework to nonlinear hyperelastic constitutive modeling for solid shells and deals with the investigation of two exemplary problems: (*i*) finite bending delamination analysis of bi-layered systems, and (*ii*) delamination of stiff islands on stretchable substrates subjected to tensile loading. The developed computational framework allows the incorporation of several issues into the computations, namely: (*i*) the implementation of three-dimensional constitutive laws for both the layer and the substrate; (*ii*) the potential of introducing geometric imperfection in the postbuckling evolution that reflects twisting deformation patterns, and (*iii*) the use of kinematically compatible layer-interface formulations for delamination analyses.

The manuscript is organized as follows. Section 2 provides an overview of the three-dimensional finite element framework, where a brief description of the solid shell and the cohesive interface formulations for finite deformation are given. The numerical investigation with respect to the finite bending analysis is presented in Section 3. The application of the proposed methodology to delamination of stiff islands (silicon-based) resting on soft substrates is addressed in Section 4, where the obtained results are compared with earlier numerical predictions and experimental data by [2] which serve as benchmark. Finally, Section 5 concludes the present study and discusses future prospective developments.

2. Computational framework

In this section, the fundamentals of the computational framework for the interplay of geometrical instabilities and delamination of layer-substrate systems undergoing finite deformation are briefly outlined. Section 2.1 addresses the main concepts associated with the solid shell used to model the thin layer, whereas the central aspects of the cohesive formulation and the hyperelastic constitutive law for the substrate and the thin layer are given in Sections 2.2 and 2.3, respectively. The variational formulation of the current framework is outlined in Section 2.4, and finally, the finite element implementation aspects of the solid shell and of the cohesive interface elements are succinctly covered in Sections 2.5 and 2.6, respectively.

2.1. Solid shell model

The thin layer of the film-substrate system is modelled by means of the solid shell concept, see Fig. 1a for a schematic representation. This concept has been extensively used in the structural modeling of thin-walled structures due to its robustness and simplicity in comparison with alternative surface-based shell formulations. Furthermore, this modeling option allows the use of three-dimensional constitute models within the shell body, preventing the standard plane-stress assumption that is classically adopted in shells, see [25,27], among others.

The shell body is parameterized in the convective space by the local natural reference system $\boldsymbol{\xi} = \{\xi^1, \xi^2, \xi^3\}$, with $(\xi^1, \xi^2, \xi^3) \in : \square = [-1, 1] \times [-1, 1] \times [-1, 1]$, where $\square$ represents the reference parent-domain cube. Let be $\mathbf{X}(\xi^1, \xi^2, \xi^3)$ the position vector of a material point in the undeformed configuration $\mathcal{B}_0$, i.e. $\mathbf{X}(\xi^1, \xi^2, \xi^3) \in \mathcal{B}_0$, while the corresponding position vector in the deformed configuration is denoted by $\mathbf{x}(\xi^1, \xi^2, \xi^3) \in \mathcal{B}_t$.

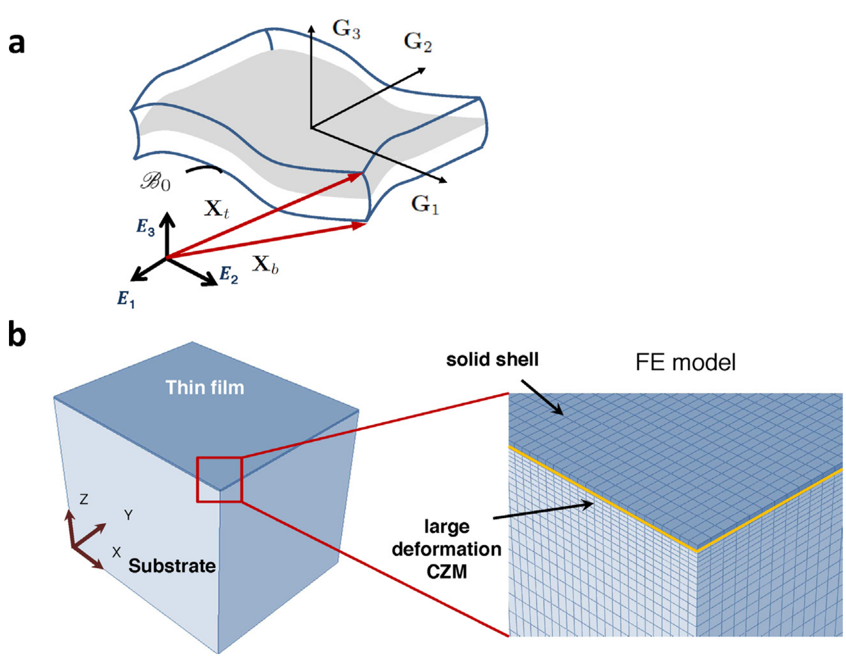

Fig. 1. (a) Solid shell parametrization of the continuum body. (b) Thin layer-substrate system including solid shell modeling for the thin layer and large deformation cohesive zone model at the interface layer-substrate for triggering delamination events.