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Integral identities for fracture along imperfectly joined anisotropic ceramic bimaterials

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ABSTRACT

We study a crack lying along an imperfect interface in an anisotropic bimaterial. A method is devised where known weight functions for the perfect interface problem are used to obtain singular integral equations relating the tractions and displacements for both the in-plane and out-of-plane fields. The problem can be considered as modelling bimaterial ceramics which are joined with a thin soft adhesive substance. The integral equations for the out-of-plane problem are solved numerically for orthotropic bimaterials with differing orientations of anisotropy and for different extents of interfacial imperfection. These results are then compared with finite element computations.

1. Introduction

Singular integral equations have played a significant role in the study of crack propagation in elastic media since their introduction by Muskhelishvili [24] and have garnered much scientific attention [31]. They have been used in the analysis of crack problems in complex domains containing an arbitrary number of wedges and layers separated by imperfect interfaces [19,20]; the resulting singular integral equations with fixed point singularities have been analysed by Duduchava [8], based on the theory of linear singular operators [10]. More recently, singular integral equations have been applied to problems involving interfacial cracks in both isotropic [18,25] and anisotropic bimaterials [22,40] and also in thermodiffusive bimaterials [21]. This paper extends the singular integral equation approach to fracture in an anisotropic bimaterial containing an imperfect interface.

Interfacial problems concerning a semi-infinite crack along a perfect interface in an anisotropic bimaterial have been considered in [33] through the use of the formalisms proposed by Stroh [32] and Lek [14]. Expressions were found for the stress intensity factors at the crack tip under the restriction of symmetric loading on the crack faces. Using weight function techniques introduced by Bueckner [6] and developed further by Willis and Movchan [39], an approach was developed to find stress intensity factors for an interfacial crack along a perfect interface under asymmetric loading for both the static and dynamic cases, see [23,28], respectively. More widely, weight functions are well developed in the literature for a wide range of fractured body geometries and allow for the evaluation of important constants that may act as fracture criteria. For instance, weight functions have been obtained for a corner crack in a plate of finite thickness [41], a 3D semi-infinite crack in an infinite body [13] and a crack lying perpendicular to the interface in a thin surface layer [9].

Imperfect interfaces provide a more physically realistic interpretation of a ceramic bimaterial than a perfect one, accounting for the fact that the interface between two materials is rarely sharp. Such interfaces are an important feature of modern functional piezoceramics. Atkinson [2] took such imperfections into account by suggesting the interface be replaced with a thin strip of finite thickness, which provided the bonding material occupying the strip is sufficiently soft may be replaced by so-called imperfect interface transmission conditions. These allow for an interfacial displacement jump in direct proportion to the traction, which is itself continuous across the interface [1,15,16]. Such transmission conditions alter physical fields near the crack tip significantly; for instance the usual perfect interface square root stress singularity is no longer present and is instead replaced by a logarithmic singularity [17], although tractions remain bounded along the interface. More recently, Wang et al. [37] investigated the joining of various engineering ceramics and composites experimentally

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and Bordignon et al. [5] modelled a shear band as a zero-thickness non-linear interface. More general imperfect interface transmission conditions were derived by Benveniste and Miloh [4] which considered a thin curved isotropic layer of constant thickness, while Benveniste [3] presented a general interface model for a 3D arbitrarily curved thin anisotropic interphase between two anisotropic solids.

Weight function techniques have been recently adapted to imperfect interface settings to quantify crack tip asymptotics in thin domains [35], analyse problems of waves in thin waveguides [34] and conduct perturbation analysis for large imperfectly bound bimaterials containing small defects [36]; the absence of the square root singularity means that the weight functions are not used to find stress intensity factors, but instead yield asymptotic constants which describe the crack tip opening displacement. This quantity was proposed for use in fracture criteria by Wells [38] and Cottrell [7] and later justified rigorously by Rice and Sorenson [29], Shih et al. [30] and Kanninen et al. [12]. Despite their great utility, the derivation of such weight functions is often not straightforward and so the approach deployed in the remainder of this paper efficiently utilises existing relationships between known weight functions without the need to derive further expressions.

The problem considered here is the anisotropic equivalent of that seen in [18], which considered solely isotropic bimaterials. Besides this, perhaps the key novel feature in the present manuscript from a methodology viewpoint, is that known weight functions derived for the *perfect* interface problem are used in the derivation of singular integral equations for the *soft imperfect* interface case. This differs from previous approaches; for instance Mishuris et al. [18] used specially derived weight functions that took into account the local crack-tip behaviour brought about by the presence of imperfect interface transmission conditions, whereas the approach employed here uses existing perfect interface weight functions, which have fundamentally different behaviour near the crack tip to the physical solution in the imperfect interface problem. The derived identities have potential applications in the modelling of piezoceramics, solid oxide fuel cells and other ceramic devices.

The paper is structured as follows: Section 2 introduces the problem geometry and model for the imperfect interface. In Section 3, previously found results used in the derivation of the singular integral equations are discussed. These include the weight functions derived using the method of Willis and Movchan [39] and the Betti formula which can be used to relate the weight functions to the physical fields along both the crack and imperfect interface. Section 4 concentrates on solving the out-of-plane (mode III) problem. Singular integral equations are derived and used to obtain the displacement jump across both the crack and interface for a number of orthotropic bimaterials with varying levels of interface imperfection. Finite element methods for the same physical problems are also used to obtain the same results and then a comparison is made between the results obtained from the two opposing methods. The in-plane problem is considered in Section 5, where singular integral equations are obtained for the mode I and mode II tractions and displacements and some computations are performed.

2. Problem formulation

We consider an infinite anisotropic bimaterial with an imperfect interface and a semi-infinite interfacial crack respectively lying along the positive and negative x_1 semi-axes. The materials above and below the x_1-axis will be denoted materials I and II respectively.

The imperfect interface transmission conditions for $x_1 > 0$ are given by

$$\mathbf{t}(x_1, 0^+) = \mathbf{t}(x_1, 0^-), \tag{1}$$

$$\mathbf{u}(x_1, 0^+) - \mathbf{u}(x_1, 0^-) = \mathbf{K}\mathbf{t}(x_1, 0^+), \tag{2}$$

where $\mathbf{t} = (t_1, t_2, t_3)^T = (\sigma_{21}, \sigma_{22}, \sigma_{23})^T$ is the traction vector and $\mathbf{u} = (u_1, u_2, u_3)^T$ is the displacement vector. The matrix $\mathbf{K}$ quantifies the extent of imperfection of the interface, with $\mathbf{K} = \mathbf{0}$ corresponding to the perfect interface. For an anisotropic bonding material, $\mathbf{K}$ has the following structure:

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & 0 \\ K_{12} & K_{22} & 0 \\ 0 & 0 & \kappa \end{pmatrix}. \tag{3}$$

The loading on the crack faces is considered known and given by

$$\mathbf{t}(x_1, 0^+) = \mathbf{p}^+(x_1), \quad \mathbf{t}(x_1, 0^-) = \mathbf{p}^-(x_1), \quad \text{for } x_1 < 0. \tag{4}$$

The geometry considered is illustrated in Fig. 1. The only restriction imposed on $\mathbf{p}^\pm$ is that they must be self-balanced; note in particular that this allows for discontinuous and/or asymmetric loadings. The symmetric and skew-symmetric parts of the loading are given by $\langle \mathbf{p} \rangle$ and $[\![\mathbf{p}]\!]$ respectively, where the notation $\langle f \rangle$ and $[\![f]\!]$ respectively denote the average and jump of the argument function:

$$\langle f \rangle(x_1) = \frac{1}{2}(f(x_1, 0^+) + f(x_1, 0^-)), \quad [\![f]\!](x_1) = f(x_1, 0^+) - f(x_1, 0^-).$$

3. Application of existing weight functions

3.1. Weight functions

Bueckner [6] defined weight functions as non-trivial singular solutions of the homogeneous traction-free problem. Willis and Movchan [39] introduced weight functions in a mirrored domain and related physical quantities with the auxiliary weight functions via use of Betti's identity; this procedure has been recently used to derive singular integral equations for isotropic bimaterials joined by an imperfect interface [18]. The approach employed there required the use of weight functions that had been designed for an imperfect interface setting for isotropic bimaterials. In the spirit of the efficiency outlined in the introduction, we will in this section introduce a method where integral identities for the physical problem with an imperfect interface are found using existing weight functions formulated in a *perfect* interface