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Data relocalization to mitigate slow convergence caused by under-resolved stress fields in computational damage mechanics

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Abstract

Nonlocal theories are often regarded as necessary to achieve mesh-insensitive predictions of ceramic damage, while purely numerical sources of abnormally slow convergence have been largely ignored. An alternative stress field regularization technique compensates for under-resolution of the stress field by preserving the probability of failure initiation regardless of whether the domain is discretized into few or many elements. This method, called data relocalization, effectively replaces the uniform stress state assigned to a low-order finite element with the m-norm of the actual (spatially varying) stress field, where m is the Weibull modulus. As an example, a ceramic Brazilian indirect tension test (for which the pre-failure stress field is known) is shown to exhibit intolerably slow convergence when run with only scale-dependent and statistically variable strength. Nearly convergent solutions are then achieved even on a very coarse mesh using data relocalization, thus proving a purely numerical (nonphysical) source of mesh sensitivity.

Keywords: Data delocalization/relocalization; Method of manufactured solutions; Brazilian test; Failure probabilities; Aleatory uncertainty

1. Introduction

Achieving mesh independent results in damage simulations has been a persistent unsolved challenge to model developers. Lab data show compellingly a size effect for strength, in which large specimens are weaker on average.[12,2] When analyzing lab data for strength, the experimentalists need to report the specimen size that goes with that strength, which is a nontrivial requirement if the stress field is not constant over the specimen. Hild et al.[17] proposed an "effective volume"[8] approach given as a stress-weighted m-norm average of the volume, where m is the Weibull modulus. Specifically, the effective volume is given by

$$V_{\text{eff}} = \int_V \left(\frac{\sigma}{\sigma_{\text{peak}}}\right)^m \mathrm{d}V, \tag{1}$$

where σ is a stress invariant (e.g. maximum principal stress), and σ_{peak} is the maximum value of σ in the domain V. This approach, called *data delocalization*,[3,4] appropriately gives high stress regions higher weight.

Conventional plasticity theories fail to capture this size effect because there is no length scale associated with the constitutive model. An alternative route to capture these size effects is to use gradient plasticity theory[20] or nonlocal theory,[10,11] but their formulations currently are based on overly simplistic constitutive models. Previous work[7] using statistical variability of strength and scale effects has considerably reduced mesh sensitivity in dynamic sphere indentation problems. The same features provided much more realistic qualitative statistical variability of strength and scale effects,[19] but failed to exhibit similar benefits with respect to convergence. Achieving convergence on failure initiation is a necessary pre-requisite to predicting mesh-insensitive failure progression.

This paper addresses a numerical technique (which from here on is termed as "data relocalization") to achieve the first requirement – convergence on failure initiation. Section 2 reviews the theory that an m-norm of the actual (spatially varying) stress field needs to be applied to the element to preserve probability of failure initiation of a finite subdomain of the body regardless of whether the domain is discretized into few or many elements. A 1D exploratory problem is designed to test this technique. Section 3 solves a 1D problem using the method of manufactured solutions in a finite-element code. With conventional methods, significant mesh dependency is demonstrated for the onset of failure using low-order shape functions, with only slight improvement using higher-order shape functions. Data relocalization, on the other hand, provides a substantial

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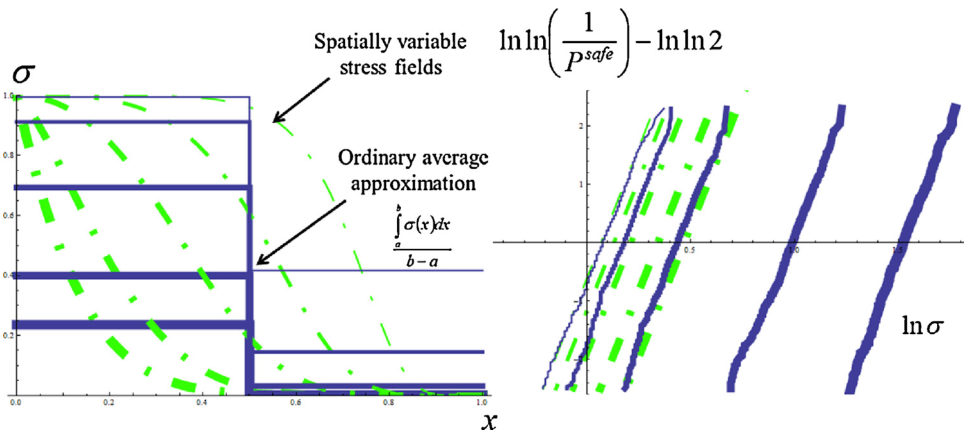

Fig. 1. The two-element exploratory problem. (Left) Green dashed lines are the known stress fields over the element. Solid blue lines are the corresponding ordinary (1-norm) average approximation over the element (i.e.,thick blue line corresponds to thick green line). (Right) Green lines are the analytical Weibull lines. Blue lines are the corresponding numerical failure probability curves. (For interpretation of the references to color in the text, the reader is referred to the web version of the article.)

improvement of convergence. Section 4 illustrates this technique on a more complicated Brazilian test problem.

2. Data relocalization

Based on classical Weibull theory[24] for a uniform stress field σ within a specimen of volume V (considered as a single-element mesh), the probability that the sample is safe from failure initiation is

$$P^{\text{safe}} = 2^{-(V/\overline{V})(\sigma/\overline{\sigma})^m}, \tag{2}$$

where $\overline{\sigma}$ is the median strength associated with a reference volume $\overline{V}$,and m is the Weibull modulus. For piecewise constant stresses, σ_1 and σ_2, applied to volumes V_1 and V_2 (considered as a 2-element mesh) covering the same total volume V, the whole volume is taken to be safe if each individual volume is safe. This assertion implicitly neglects spatial correlation of strength, which is appropriate if the volumes V_1 and V_2 are large in comparison to imperfections such as microcracks that give rise to failure. Accordingly, the probability that the whole volume V is safe is

$$P^{\text{safe}} = 2^{-(V_1/\overline{V})(\sigma_1/\overline{\sigma})^m} 2^{-(V_2/\overline{V})(\sigma_2/\overline{\sigma})^m}. \tag{3}$$

The goal is to preserve probability of failure initiation. Setting Eqs. (2) and (3) equal and solving for σ, we have

$$\sigma^m = \frac{V_1{\sigma_1}^m + V_2{\sigma_2}^m}{V}, \tag{4}$$

which implies

$$\sigma = \left(\frac{V_1{\sigma_1}^m + V_2{\sigma_2}^m}{V}\right)^{1/m}. \tag{5}$$

Extending the concept to continuous fields,

$$\sigma_{\text{eff}} = \left(\frac{\int_V \sigma^m \, \mathrm{d}V}{V}\right)^{1/m}. \tag{6}$$

This is the effective uniform stress that would need to be applied to a specimen of volume V to produce the same probability of failure as the actual non-uniform stress field on V. This is an important observation because low-order shape functions effectively treat stress as constant over each element. If the actual non-uniform stress field is known, we will demonstrate that Eq. (6) gives an artificially intensified uniform stress that preserves the failure probability. Rather than changing the stress itself (which would inappropriately change elastic strain), we use Eq. (6) to artificially reduce the element strength based on our assumed knowledge of the exact non-uniform stress field over the element domain. We call this approach *data relocalization* because it accounts for the stress concentrations that cannot be resolved on the grid, essentially reversing the data delocalization in Eq. (1). Of course, an exact pre-failure stress field is not known in practice, so this work is merely serving as a demonstration that under-resolved stress fields caused by low-order shape functions clearly contribute to mesh sensitivity of failure.

To explore the consequences of a piecewise-constant stress representation, a problem was designed having a large stress gradient. The new method to compensate for under-resolved stress fields was tested using an exploratory problem under the assumption that we know the actual pre-failure stress field over a specimen *a priori*. The following sequence of steps explain the method to draw Weibull plots from strength testing data (whether acquired though actual or virtual experiments).

1. Obtain a table of measured stresses at failure initiation: $\{\sigma_1, \sigma_2, \sigma_3, \ldots \sigma_n\}$.
2. Sort the failure stresses: $\sigma_1 < \sigma_2 < \sigma_3 \ldots < \sigma_n$.
3. Set $\overline{\sigma}$ to be the median of the data.
4. Create a table of abscissa values defined by $x_k = \log\left(\frac{\sigma_k}{\overline{\sigma}}\right)$.
5. For the k^{th} point in a set of measured data points, set $P_k^{\text{safe}} = 1 - ((k - (1/2))/n)$. This is a conventional complementary cumulative distribution estimator,[15] which we have found gives the best accuracy for small data sets.
6. Create a table of ordinate values defined by $y_k = \log\log(1/P_k^{\text{safe}}) - \log\log 2$.
7. Plot y_k vs. x_k, which is the Weibull plot.