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Short Communication

Sintering force behind shape evolution by viscous flow

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Abstract

A non-spherical amorphous/glass particle relaxes to its equilibrium shape by viscous flow driven by capillarity at elevated temperatures. The shape evolution of an axisymmetric ellipsoidal particle, which is a model of viscous sintering of two spheres in the later stage, was simulated by the finite-element method. A force inside the particle is defined on a plane which cuts the center and perpendicular to the semi-major axis. It is given by the surface tension along the circumference and the average pressure on the plane. The velocity field near the center of the particle is a pure straining motion. The simulation showed that the radial growth rate was proportional to a stress which is given by the force divided by the cross-sectional area. This result suggests that the force can be regarded as the sintering force. Conversely, the force can be determined from the experimental observation of the length of semi-minor axis.

Keywords: Sintering; Micromechanical modeling; Simulation; Viscous flow; Particle

1. Introduction

From a mechanical point of view, sintering is a deformation of porous materials driven by a thermodynamic quantity, which is called sintering stress.[1–4] In a macroscopic definition, the sintering stress is given by the free strain rate and the bulk viscosity of the porous body.[1] This approach is useful for understanding the stresses generated by constraints, such as rigid inclusions or attachment to a substrate. In a microscopic definition, many attempts have been made to understand the sintering stress from forces in an aggregate of a huge number of particles.[5–7] In any sintering problem of practical interest, the particles are in contact with several neighbors, so the stress field will be complicated and, generally anisotropic. The detailed microstructural information is available from the direct observation,[8] X-ray synchrotron microtomography,[9] and numerical simulations.[10] The microscopic approach is used to analyze the effect of powder processing on the particle packing and the anisotropy in macroscopic sintering stress tensor.[11]

In sintering by grain boundary diffusion, the relative velocity between two particles is proportional to a thermodynamic force, i.e., the sintering force, which is defined as a force acting on contact.[12] In viscous sintering,[13] the sintering force has been studied also for the coalescence of glass particles.[14] Aggregates of glass particles change their shape by viscous flow driven by capillarity at elevated temperatures. The evolution of particle shape is simulated numerically from the principles of fluid mechanics,[15–17] for example, by using the finite element method.[18–20] Viscous sintering of multiparticles has been simulated by several models[21–24] recently. However, these results have not been analyzed by using the concept of sintering force. As the simplest example of viscous sintering, the present paper shows that a force can be defined even inside an evolving single particle. The shape evolution is described as a response to the force, and it can be regarded as the thermodynamic driving force for sintering.

2. Numerical analysis

Viscous sintering occurs by material flows driven by capillarity. The stress in a viscous fluid is expressed by

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (1)$$

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where u_i is the velocity, μ is the viscosity, $p = -\sigma_{ii}/3$ is the pressure. We consider a very small particle with high viscosity, then, the deformation of the particle is described by Stokes equation[25]

$$\frac{\partial p}{\partial x_i} = \mu \frac{\partial^2 u_i}{\partial x_k \partial x_k}. \tag{2}$$

The mass conservation in incompressible flow is expressed as

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{3}$$

The summation convention for repeated indices is applied throughout this paper. The boundary condition on the surface is

$$-pn_i + \mu \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) n_k = \gamma_s \kappa n_i \tag{4}$$

where γ_s is the surface energy, n_i is the unit (outward) normal to the surface, and $\kappa = \mathrm{div}\,\mathbf{n}$ is the curvature. The curvature is defined that it is negative for a spherical particle.

3. Results

We consider the viscous flow in an isolated spheroidal particle which has semi-axes $a = b$, c as shown in Fig. 1. The radius of the equivalent sphere is $r_0 = (abc)^{1/3}$. The deformation of an axisymmetric ellipsoid (prolate spheroid, initial axial ratio, $c/a = 3$) was simulated by using the finite element package ANSYS, Polyflow (Ver. 14.5). We used an axisymmetric model with 2673 elements. The total of 5000 time steps were taken. The axial lengths are plotted as functions of the dimensionless time $t^* = \gamma_s t/r_0\mu$ in Fig. 1. The elongated particle becomes a sphere by viscous flow; that is the spheroidization.

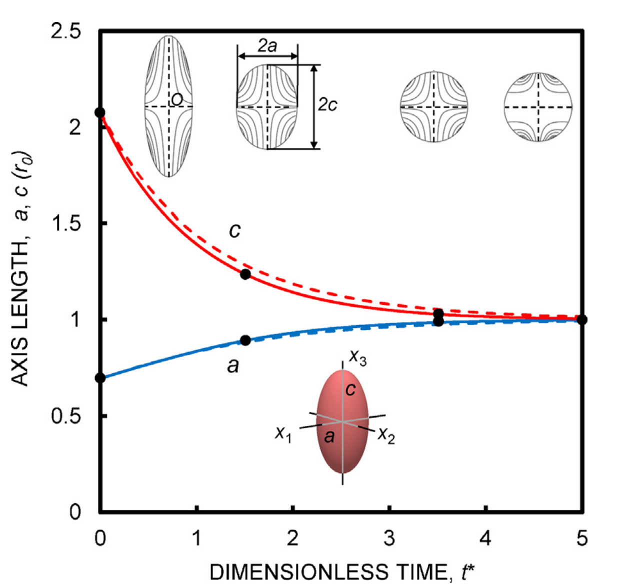

Fig. 1. Spheroidization of an axisymmetric ellipsoid (initial axial ratio $c/a = 3$). Solid lines show semi-axes of the spheroidal particle. The broken lines show the prediction by the tensor-virial equation [26]. Evolving particle shape is shown along with stream lines. The zero stream function is defined along symmetry planes and is indicated by dashed lines.

The cross-section of the particle in Fig. 1 shows the streamline, which is a line of flow whose tangent is everywhere parallel to velocity. The streamlines indicate that material flows so as to make the elongated particle to be more spherical. The flow field has axial symmetry. The velocity distribution shows that the center of the ellipsoid O is a stagnation point, where velocity is zero. The velocity field near O is a pure straining motion[21]

$$u_i = \dot{\varepsilon}_{ij} x_j \tag{5}$$

where $\dot{\varepsilon}_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$ is the velocity gradient tensor, or strain rate tensor. Eq. (1) is rewritten as

$$\dot{\varepsilon}_{ij} = \frac{\sigma'_{ij}}{2\mu} \tag{6}$$

where $\sigma'_{ij} = \sigma_{ij} + p\delta_{ij}$ are the deviatoric stress components.

Here we define a force F acting on a symmetry plane, which cuts the center O and perpendicular to the x_3-axis in Fig. 1

$$F = -\bar{p}A + \gamma_s L \tag{7}$$

where $A = \pi a^2$ is the cross sectional area, $L = 2\pi a$ is the circumference, and $\bar{p}$ is the average pressure on the plane

$$\bar{p} = \frac{1}{A} \int_A p dS. \tag{8}$$

The pressure distribution inside the particle is non-uniform as shown in Fig. 2. The pressure on the cross-section depends on the surface curvature $\kappa = 1/a + 1/r'$ at the circumference, where r' is the radius of curvature in x_1–x_3 plane. The average pressure decreases initially with increasing the length of the semi-minor axis a, then, the pressure increases to the equilibrium pressure $2\gamma_s/r_0$ as r' decreases.

The force is plotted as a function of the dimensionless time in Fig. 3. The force reaches a maximum at $t^* = 0.5$, and decreases to zero at equilibrium.

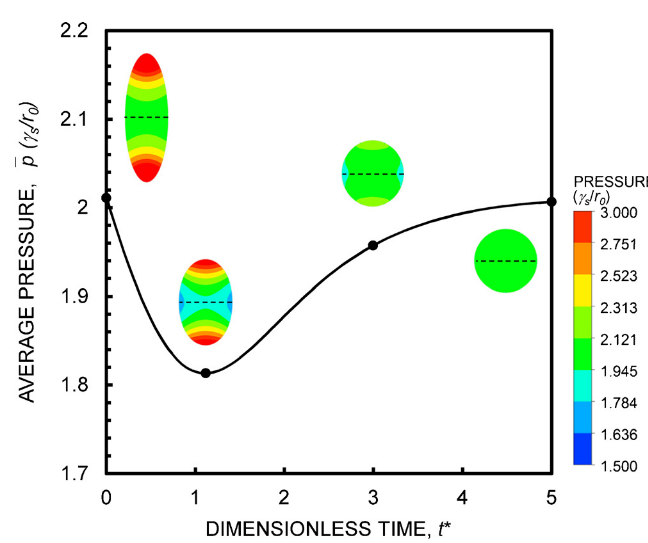

Fig. 2. The average pressure $\bar{p}$ on a symmetry plane. The particle shape is shown along with pressure distribution. Dashed line shows the symmetry plane.