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Journal of the European Ceramic Society 34 (2014) 1547–1552

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Evaluation of the complex material constants of piezoelectric ceramics in the thickness vibration mode

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Received 31 July 2013; received in revised form 10 December 2013; accepted 16 December 2013
Available online 14 January 2014

Abstract

A new noniterative method, for determining the dielectric, piezoelectric and elastic constants, in complex form, for piezoceramic materials, in the thickness extensional mode, was proposed.

This method is very flexible, as it is based on the standard approach to determine the elastic stiffness, followed by the measurement of the impedance at two frequencies to calculate the dielectric and piezoelectric constants, by solving a system of two equations.

The new method was tested on materials with various thickness coupling factors (k_t = 4.5–60%) and mechanical quality factors (Q_{mt} = 20–1600), proving very good accuracy for all of them.

The accuracy of standard method was also evaluated for the same materials.

Keywords: Complex material constants; Noniterative method; Piezoelectric ceramics; Thickness mode; Method accuracy

1. Introduction

Characterization of piezoelectric and electromechanical properties of piezoelectric ceramics is performed by determining their piezoelectric, dielectric and elastic constants, as well as their electromechanical coupling factors, either as real or complex quantities, depending on whether their losses are ignored or taken into account.

The standard method, described in the latest version of the IEEE standard,[1] ignores the losses, assuming that all constants are real quantities. This standard, which is a revision of the old ones, is at present outdated and withdrawn because of its limitations. Hereby the need of alternative characterization methods, that take into account the losses, treating the material constants as complex quantities, according to experimental spectra of the electrical impedance. These methods are based on iterative and noniterative, algorithms, fitted for determining the material constants specific to each vibration mode of piezoceramic resonators. An accurate evaluation of all these material constants including their losses is required for the three dimensional modelling and design of piezoceramics and their applications by numerical methods as Finite Element Analysis.[2,3] For thickness extensional mode, there are several iterative[4–7] and noniterative methods,[8] which provide material constants in complex form. In a previous paper,[9] a new noniterative method was proposed for calculating the material constants, in complex form, for the radial mode. The algorithm is very simple and easy to apply, providing a very good accuracy for a wide range of piezoceramic materials.

In the present paper, this new method was developed and implemented for the thickness mode. The errors of the complex material constants, obtained by this method, were determined for materials with various thickness coupling factors (k_t = 4.5–60%) and mechanical quality factors (Q_{mt} = 20–1600).

2. Measurements

The thickness mode of vibration is generated in a disc shaped piezoceramic resonator, by applying a sinusoidal voltage, with variable frequency, provided by an impedance analyzer (HP-4294A). The impedance spectra (resistance R and reactance X) of the piezoresonator are measured, as a function of frequency, within the parallel resonance band of the fundamental thickness mode and stored as input resonance spectra, to be compared with the output spectra (generated with the constants provided by this method).

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0955-2219/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jeurceramsoc.2013.12.033

The parallel resonance frequency f_p corresponding to R_{max}, the maximum of R, and the frequencies $f_{X\,max}$ and $f_{X\,min}$ of maximum and minimum of X around f_p, respectively, are determined from these spectra and stored. Electrical impedance, is also measured at two frequencies f_{1sis} and f_{2sis}, outside the resonance band, and the corresponding values Z_{1sis} and Z_{2sis} are stored. The criteria used to select these frequencies are discussed in Section 3.

3. Description of the method

The method is based on the following expression[5,6] of the electrical impedance Z, of a disc piezoceramic resonator oscillating in the thickness mode, derived from Eq. (105), p. 230 of[10]:

$$Z = -i\frac{t}{2\pi f A \varepsilon_{33}^S} + i\frac{h_{33}^2\ \tan(\pi f t\sqrt{\rho/c_{33}^D})}{2\pi^2 A f^2 c_{33}^D \sqrt{\rho/c_{33}^D}} \quad (1)$$

where f, A, t and ρ are the frequency, electrode area, thickness and density of the sample, respectively and $i=\sqrt{-1}$. The dielectric permittivity at constant strain ε_{33}^S, the piezoelectric constant h_{33}, and the elastic stiffness at constant dielectric displacement c_{33}^D are complex quantities, which have to be determined by this method. The first step is to calculate the elastic stiffness c_{33}^D by:

$$c_{33}^D = c_{33\ st}^D \left[1 + i\frac{\Delta f}{f_p}\right] \quad (2)$$

where the real part, $c_{33\,st}^D$, represents the elastic stiffness of a lossless resonator, given by the following formula:

$$c_{33\ st}^D = 4\rho t^2 f_p^2 \quad (3)$$

The imaginary part of c_{33}^D is determined with:

$$\Delta f = f_{X\,min} - f_{X\,max} \quad (4)$$

After substituting the values of c_{33}^D, Z_{1sis} and Z_{2sis} with their corresponding frequencies f_{1sis} and f_{2sis} into Eq. (1), a system of two equations linear with respect to the constants $1/\varepsilon_{33}^S$ and h_{33}^2 is obtained. By solving it, material constants ε_{33}^S and h_{33}^2 are determined. The frequencies $f_{1,2sis}$, used to calculate the dielectric and piezoelectric constants are symmetrically situated outside the resonance band ($f_{1sis} < f_m < f_n < f_{2sis}$), where f_m and f_n are the frequencies corresponding to minimum and maximum of absolute impedance, respectively, in the fundamental resonance band of the thickness mode. The frequencies $f_{1,2sis}$ should be chosen such that the absolute impedance has about the same values at these frequencies. For materials with very low k_t and Q_{mt}, the difference between them ($f_{2sis} - f_{1sis}$) is empirically recommended to be about ten times the difference ($f_n - f_m$), since otherwise they would be to close to f_m and f_n, where the dielectric constant is determined with low accuracy. At parallel resonance, which corresponds to the thickness mechanical resonance of the disc,[5,11] a large amount of input energy is converted to elastic energy, thus allowing an accurate determination of the elastic constant around f_p, to the detriment of the dielectric constant. Therefore, it is necessary to determine the dielectric constant far from f_p, where the dielectric energy becomes dominant. This is the reason of choosing the frequencies $f_{1,2sis}$, near (to provide the piezoelectric constant with good accuracy) but outside the resonance band.

4. Results and discussion

The new method was tested on simulated resonance spectra corresponding to materials with various thickness coupling factors (k_t = 4.5–60%) and mechanical quality factors (Q_{mt} = 20–1600). These spectra, playing the role of the experimental ones, were generated by substituting the input complex values of the constants ε_{33}^S, h_{33} and c_{33}^D, into Eq. (1). The input constants given in Table 1, were chosen to correspond to nine

Table 1
Input constants of materials 1–9.

Material	Input constants							
	$\varepsilon_{33}^S/\varepsilon_0$	h_{33} (10^8 V/m)	c_{33}^D (10^{10} N/m^2)	k_t	Q_{mt}	$f_p/\Delta f_p$	$f_p/\Delta f_{hb}$	ρ (kg/m^3)
1. k_t = 0.60 Q_{mt} = 100	$700 - i\,5$	$30 + i\,0.1$	$15 + i\,0.15$	$0.61 - i\,0.0032$	100	100	100	7650
2. k_t = 0.58 Q_{mt} = 500	410−i	$37.5 + i\,0.07$	$15 + i\,0.03$	$0.58 - i\,0.0029$	500	500	500	7600
3. k_t = 0.39 Q_{mt} = 300	$130 - i\,3$	$45 + i\,0.5$	$15 + i\,0.05$	$0.39 - i\,0.00082$	300	300	300	7550
4. k_t = 0.25 Q_{mt} = 1600	$110 - i\,0.59$	$30 + i\,0.08$	$13.8 + i\,0.0086$	$0.25 - i\,0.0019$	1604	1604	1604	7540
5. k_t = 0.42 Q_{mt} = 26	$330 - i\,15$	$28 + i\,0.26$	$13.2 + i\,0.5$	$0.42 - i\,0.013$	26	26	26	5300
6. k_t = 0.20 Q_{mt} = 20	$235 - i\,5$	$10 + i\,0.3$	$5 + i\,0.25$	$0.20 - i\,0.0011$	20	21	20	6000
7. k_t = 0.10 Q_{mt} = 20	$300 - i\,10$	$6.8 + i\,0.25$	$12 + i\,0.6$	$0.10 - i\,0.0005$	20	26	20	7650
8. k_t = 0.09 Q_{mt} = 22	$185 - i\,8$	$5.3 + i\,0.1$	$5.7 + i\,0.26$	$0.09 - i\,0.0023$	22	28	22	6000
9. k_t = 0.045 Q_{mt} = 200	$220 - i\,10$	$6.5 + i\,0.008$	$40 + i\,0.2$	$0.045 - i\,0.0011$	200	203	199	6500