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Direct numerical simulations on crack formation in ceramic materials under thermal shock by using a non-local fracture model

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Abstract

In this work, a non-local failure model was proposed and implemented into a finite element code. It was then used to simulate the crack evolution in ceramic materials subjected to thermal shock. By using this numerical model, the initiation and propagation of cracks in water quenched ceramic specimens were simulated. The numerical simulations reproduced faithfully the crack patterns in ceramic specimens underwent quenching tests. The periodical and hierarchical characteristics of the crack patterns were accurately predicted. The numerical simulations allow a direct observation on whole the process of crack initiation and growth, which is quite a difficult task in experimental studies. The failure mechanisms and the fracture procedure are discussed according to the numerical results obtained from the simulations. It is shown that the numerical model is simple, robust, accurate and efficient in simulating crack evolution in real structures under thermal shock.

Keywords: Thermal shock; Crack patterns; Non-local fracture model; Numerical simulations

1. Introduction

Ceramic materials are widely used in various industries due to their excellent high temperature mechanical properties, corrosion resistance, wear resistance, erosion resistance, oxidation resistance, etc. However, their inherent brittleness and insignificant ductility make them particularly vulnerable to thermal shock failure. In general, crack formation is considered as the major reason of failure in thermo-structural engineering. Understanding the mechanisms of cracking process in ceramics under thermal loads has been one of the most importance tasks in the research of this field.

Researches on fracture of ceramic materials underwent thermal shock was initiated 60 years ago by Kingery.[1,2] He first proposed a so-called "critical stress" fracture criterion according to which cracks appear when the maximal thermal stress reaches the ultimate stress of the material. Hasselman proposed a different approach in which the driving force for crack propagation is derived from the elastic energy stored in the body at the instant of fracture.[3]

Afterward, numerous theoretical and experimental studies on thermal shock failure of ceramics have been reported.[4–18] Among them, Hasselman gave qualitatively theoretical predictions of crack propagation behaviour in polycrystalline alumina rods under thermal shock by water quenching.[5] Lu and Fleck analyzed the thermal shock resistance of brittle solids by use of a stress-based fracture criterion.[9] Bažant et al. and Nemat-Nasser et al. studied the stability of propagation of thermal shock cracks by using the energy principle, theoretically discussed the length hierarchy phenomenon of the crack patterns.[12–14] This phenomenon was also studied theoretically and experimentally by Bahr et al.[15–17] Jenkins has calculated the spacing and penetration of cracks by using a method based on energy minimization.[18]

Recently, Jiang et al. carried out experimental and numerical works in determining the crack patterns by taking the temperature-dependence of the material parameters into account.[19] On the basis of a variational approach, Bourdin et al. developed a variational model capable to perform complicated fracture analysis in brittle materials under thermal shock.[20]

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The crack pattern formation under thermal shock is quite a rapid and highly complex process. This process is difficult to capture with available experimental techniques. Only final crack patterns can easily be observed. This is why direct numerical simulations are particularly interesting in reproducing the cracking process. As a result, the failure mechanisms and the control parameters can be better understood. However, the direct numerical simulations have rarely been reported in the literature so far due to the inherent complexities in multi-cracking modelling. This is the main motivation of the present work.

In the previous works, the non-local and strain gradient schemes were used to model the damage and fracture in brittle materials.[21–24] However, multi-cracking phenomenon such as that observed in thermal shock experiments is always an intrinsically difficult problem to deal with. In this paper, a non-local failure criterion was proposed and implemented into a finite element code. It was then applied to simulate the crack evolution in ceramic materials subjected to thermal shock. The proposed fracture model is equivalent to the maximum principal stress criterion for a specimen under pure tensile loading, and to the Griffith–Irwin criterion for crack propagation. Consequently, this non-local fracture model can both predict crack initiation as well as crack growth. The multi-cracking problems can easily be dealt with in a natural manner. Applying to the thermal shock problems, the proposed method successfully reproduces the crack patterns in ceramic specimens after quenching. The periodical and hierarchical characteristics of the crack patterns are predicted with satisfactory accuracy. Moreover, the direct numerical simulations faithfully describe whole the cracking process, including the crack initiation, crack growth and crack arrest during quenching tests. The numerical results are presented together with the previous experimental results. The comparison permits to evaluate the accuracy and efficiency of the proposed model. Finally, we give some concluding remarks and directions to follow in future works.

2. Non-local damage model and numerical resolution method

2.1. Non-local fracture criterion

We first outline the non-local fracture model used in this work. The basic idea of this model consists in replacing the local damage driving force, an effective stress σ_e for example, by its weighted average over a representative volume V:[25]

$$\tilde{\sigma}_e(x) = \frac{1}{\int_V \alpha(x)dV} \int_V \alpha(x-y)\sigma_e(y)dy \tag{1}$$

where α is a weighting function. In the present work, a cone-shape function is adopted for simplicity:

$$\alpha(r) = \begin{cases} 0 & r > R \\ 1 - \dfrac{r}{R} & r \le R \end{cases} \tag{2}$$

where $r = ||x-y||$; R is the radius of non-local action, representing a material characteristic length which defines the size of interaction zone in failure process.

We assume reasonably that the failure in ceramic materials under uniform stress fields obeys the maximum principal stress criterion. However, this criterion cannot directly be utilized to predict crack growth due to the stress singularity near the crack tips. To overcome this shorthand, we relate the maximum principal stress criterion to a crack growth criterion throughout a non-local formulation such like Eq. (1). Thus, the non-local maximum principal stress criterion can be written as follows:

$$D = \begin{cases} 0 & \tilde{\sigma}_1 < \sigma_c \\ 1 & \tilde{\sigma}_1 \ge \sigma_c \end{cases} \tag{3}$$

where D is the damage, σ_c is the ultimate stress of the material, $\tilde{\sigma}_1$ is the non-local first principal stress. We enforce the validity of criterion (3) in two special cases: First, it should be valid in the case of a uniform tensile load. It is clear that in this case, $\tilde{\sigma}_1 = \sigma_1$, and consequently, criterion (3) is equivalent to the maximum principal stress criterion. Second, it should be valid for the growth of a mode-I crack. To this end, we assume that the near-tip stress field is governed by the Williams asymptotic expansion.[26] Therefore, for a mode-I loaded crack, the non-local first principal stress near the crack tip writes, according to (1) and (2):

$$\tilde{\sigma}_1(r,\theta) = \frac{1}{\int_0^R \int_{-\pi}^{\pi}(1-(r'/R))r'dr'd\theta'} \int_0^R \int_{-\pi}^{\pi} \times \left(1 - \frac{r'}{R}\right)\sigma_1 r'dr'd\theta' \tag{4}$$

$$\text{with} \quad \sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\left(1 + \left|\sin\frac{\theta}{2}\right|\right)\cos\frac{\theta}{2}.$$

In these expressions, K_I is the stress intensity factor, r and θ are the polar coordinates with the origin at the crack tip. Under mode I loading, the maximum non-local principal stress is located at a point on the crack axis near the crack tip $r = r_0$, $\theta = 0$ due to the symmetry. We assume that r_0 is small such that the stress at its vicinity is still governed by the crack-tip asymptotic field.

On the one hand, according to the damage criterion (3), the element at ($r = r_0$, $\theta = 0$) is broken when $\tilde{\sigma}_1 \ge \sigma_c$. On the other hand, from the Griffith–Irwin criterion of fracture, the crack grows when $K_I \ge K_{Ic}$, where K_{Ic} is the critical stress intensity factor.[27,28] This condition permits us to determine the non-local action radius R and the location of the most loaded point ($r = r_0$, $\theta = 0$) by resolving numerically the following equation:

$$f(R) = \sigma_c - \max_{r_0} \int_0^R \int_{-\pi}^{\pi} \frac{3}{\pi R^2}\left(1 - \frac{r'}{R}\right)\frac{K_{Ic}}{\sqrt{2\pi r}} \times \left(1 + \left|\sin\frac{\theta}{2}\right|\right)\cos\frac{\theta}{2} r'dr'd\theta' = 0 \tag{5}$$

with

$$r = \sqrt{(r_0 + r'\cos\theta')^2 + (r'\sin\theta')^2} \quad \tan\theta = \frac{r'\sin\theta'}{r_0 + r'\cos\theta'}$$

The geometrical quantities in Eq. (5) are shown in Fig. 1. Thus, the non-local damage criterion (3) is exactly equivalent to