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Evaluation of residual curvature in two-point bent glass fibers

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ABSTRACT

In the presence of water vapor, oxide glasses exhibit faster surface stress relaxation than bulk stress relaxation at a temperature below the glass transition temperature. When surface stress relaxation takes place in a glass under an applied stress, surface residual stress can develop upon the release of the applied stress. This phenomenon can be explored by studying the permanent bending of a fiber that is observed when a fiber is heat-treated in air under a bending load, and subsequently released from the bending load. Simple and accurate methods to evaluate the residual curvature in a bent fiber from two-point bending treatments were developed and applied to the bending of SiO_2 glass optical fibers.

1. Introduction

It was found that surface stress relaxation of silica glasses can take place at much lower temperatures than the glass transition temperature in a moist atmosphere [1–4], while bulk stress relaxation of the glasses, in general, requires a heat-treatment at the glass transition temperature or higher temperature. The surface stress relaxation can be demonstrated by subjecting a glass fiber to a bending stress at a temperature below the glass transition temperature and observing a permanent bending of the fiber upon release of the stress at room temperature.

A permanent bending of glass fibers with various compositions at temperatures much lower than their glass transition temperature was also reported earlier, including various fluoride, oxide, and chalcogenide glasses [5–10]. In these earlier works, most investigators subjected glass fibers to a bending stress by winding the fibers around a cylindrical object and attributed the phenomenon to viscous flow of the entire fibers, instead of the surface stress relaxation. In our work it was found that the bending of glass fibers can take place extensively in moist atmosphere but hardly in dry atmosphere. Furthermore, the source of the bending of the fiber was found confined to the surface layer of the fiber; the permanently bent fiber gradually straightened with successive etching of the fiber surface with an HF solution at room temperature. For these reasons the observed fiber bending was attributed to stress relaxation confined to the surface layer of the fiber [3]. The kinetics of surface stress relaxation of various glasses including silica, soda-lime, and various alkali aluminosilicate were evaluated by measuring the radius of curvature of the two-point bent fibers as a function of bending treatment time and temperature in various water vapor pressures and comparing the results with the developed theory [4].

It was found that the process was diffusion-controlled, with the reciprocal radius of curvature of the bent fiber increasing with the square root of the treatment time, most likely involving water diffusion into the glass fiber surface. The evaluation of the radius of curvature of the fiber was the most important step in this analysis [4]. Furthermore, in view of the importance of water vapor, two point bending [11] where the entire stressed region of fiber can be exposed a constant water vapor, is superior to the fiber wound around a cylindrical object where one side of the fiber is directly contacting the cylinder, obstructing the glass–water vapor interaction. On the other hand, the evaluation of the radius of curvature of two-point bent fibers is more difficult compared with a circularly bent fiber since the radius of curvature of the former is dependent on the position of the fiber. In the present paper, simple and accurate methods to evaluate the radius of curvature of two-point bent glass fibers will be presented.

When surface stress relaxation takes place while a glass fiber is under an applied load, the glass acquires a residual stress upon the release of the applied load. The residual stress developed in a glass surface can affect the mechanical strength of the glass. In fact, the residual stress development by the surface stress relaxation can be used as a means to strengthen glasses above their intrinsic strength, as will be reported in our separate paper [12]. Furthermore, it appears [4] that the surface stress relaxation and the resulting residual stress are involved in the fatigue limit of glasses [13–17], the lowest tensile stress which can cause the static fatigue or slow crack growth, as well as the coaxing effect of glass, strengthening by an application of sub-critical tensile stress [18,19]. Theoretically, it is easier to approximate the bending of a two-point bent fiber by a circularly bent fiber with a single radius. It was found that this approximation was acceptable in most cases [4]. In the present paper, however, an exact

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method of evaluating the radius of curvature of the two-point bent fiber after releasing the bending load was developed, and the method was applied to the bending of silica glass optical fiber.

When a glass fiber with a Young's modulus, E, and radius, r_0, is bent inside a tube with inner diameter, d_i, the maximum bending stress, σ_{max}, is given [11] by

$$\sigma_{max} = \frac{2.396 E r_0}{d_i - 2r_0}. \tag{1}$$

When a fiber with the radius, r_0, is subjected to a maximum bending stress, σ_{max}, which is produced by a radius of curvature, $R_0 = \frac{E r_0}{\sigma_{max}}$, for a period of time, t, the surface layer with thickness $z \ll r_0$ of the fiber can undergo complete surface stress relaxation. Then, upon the release of the bending load, when the fiber springs back to the radius of curvature R, the ratio of the two radii of curvature was found under the condition that $z \ll r_0$ [4] by

$$\frac{R_0}{R} = \frac{4z}{r_0}. \tag{2}$$

As this equation results from the considerations of the zero moment upon the release of bending loads, a condition that must be met at every position along the fiber length, all positions experiencing the same depth of surface stress relaxation z will also possess the same value of the ratio R_0/R. The depth of stress relaxation, z, was found to increase linearly with the square root of time, indicating a diffusion controlled process [4].

Previously, single overall values of the radii of curvature, R_0 and R, before and after the release of bending loads, were determined by approximating the fibers to have circular shapes [4] and the corresponding diffusion depths, z, and diffusion coefficients, D, were determined, where $z = \sqrt{Dt}$. In the present paper, an exact method which considers a radius of curvature, R_0, that varies with position, is used to evaluate the position-dependent radius of curvature, R, upon release from the bending load, for silica glass fibers in a two-point bending configuration. The obtained curvature results are compared with the earlier result based upon the circular approximation.

2. Theory

Four different methods of evaluating the radius of curvatures of the two-point bent fibers will be described here. They are 1) from the angle of the fiber ends, where the bending stress is zero, with respect to the original straight fiber, 2) from the position of the fiber where the bending stress just becomes zero, 3) from the coordinates of the bent fibers and 4) approximation of the two-point bent fiber with a circularly bent fiber.

Fig. 1 shows a schematic diagram of a two-point bent fiber under a bending stress and after release. The letter s represents a variable length along the fiber axial direction from the origin corresponding to the vertex of the fiber, and L is the total fiber length from the point of the maximum bending stress to the point where the bending stress just becomes zero. The letters, a, b and a′, b′ represent the values of the x, y coordinates of the fiber position, at s = L, before and after the release of the bending stress. The letters $\theta_0(s)$ and $\theta(s)$ represent the angles of the radius of curvature vector, (directed outwards the center of the curvature) at point s along the fiber, with respect to the vertical line (y-axis) during the application of a bending load and after the release of the bending load, respectively. Using this diagram, accurate methods of evaluating the ratio of the radii of curvature of the fiber before and after the release from the bending load, R_0/R, and in turn the depth, z, of surface stress relaxation by water diffusion, will be described.

2.1. R_0/R from the fiber angle at end point s = L

Expressing all angles in radians, the fiber length element, *ds*, is given by $ds = R_0 d\theta_0 = R d\theta$ where the ratio, R_0/R, is independent of the fiber position, s, whether indicated by θ, or θ_0, even though the values of R and R_0 are dependent on the fiber position.

And in general, $\int_{\theta(s=0)}^{\theta(s)} d\theta = \frac{R_0}{R}\int_{\theta_0(s=0)}^{\theta_0(s)} d\theta_0$ where both $\theta_{(s=0)}$ and $\theta_{0(s=0)}$ are π/2, such that

$$\theta(s) = \frac{\pi}{2} + \frac{R_0}{R}\left(\theta_0(s) - \frac{\pi}{2}\right). \tag{3′}$$

Since at s = L, $\theta_{0(s=L)} = \pi$

$$\frac{R_0}{R} = \frac{\theta_{(s=L)}}{\pi/2} - 1 = \frac{\Phi_{(s=L)}}{\pi/2}. \tag{3}$$

Upon relaxation and unloading, from the measurement of $\Phi_{(s=L)}$, which is the angle at s = L of the fiber from the vertical line (y-axis), one can determine the ratio of R_0/R. In this way, R_0/R can be most simply and directly determined as $\Phi/(\pi/2)$. Since the two-point bent fiber is straight for all points beyond s = L, the angle can be measured at any point beyond s = L.

2.2. R_0/R from the fiber coordinates at L

The ratio of radii of curvature of a two-point bent fiber before and after the load is released can be evaluated by measuring the coordinates, a′ and b′ at the point s = L upon the removal of the bending load. They can be evaluated as

$$a' = -\int_0^L \cos\theta\, ds \quad \text{and} \quad b' = \int_0^L \sin\theta\, ds.$$

According to Matthewson et al. [5],

$$ds = 0.381 L \frac{d\theta_0}{\sqrt{\sin\theta_0}}$$

Replacing θ in the above integrals using Eq. (3′) and using two-angle trigonometric identities

$$\frac{a'}{L} = 0.381\int_{\pi/2}^{\pi} \frac{\sin\left(\frac{R_0}{R}\left(\theta_0 - \frac{\pi}{2}\right)\right)}{\sqrt{\sin\theta_0}} d\theta_0 \quad \text{and}$$

$$\frac{b'}{L} = 0.381\int_{\pi/2}^{\pi} \frac{\cos\left(\frac{R_0}{R}\left(\theta_0 - \frac{\pi}{2}\right)\right)}{\sqrt{\sin\theta_0}} d\theta_0.$$

These integrals were evaluated numerically as a function of R_0/R and the results are shown in Fig. 2, with simple curve fit relations over the values of $0 < R_0/R < 0.5$ where the assumption of $z \ll r_0$ is met. When the coordinates, a′ or b′ corresponding to L can be measured, the corresponding value of R_0/R and in turn z can be determined. It is particularly striking that the values of R_0/R within this range are essentially directly equal to a′/L.

2.3. R_0/R from measuring x′ and y′ down the fiber

Coordinates, x′ and y′, at various distances s along the length of the fiber released from the bending load can be evaluated as follows:

$$x' = -\int_0^s \cos\theta\, ds \quad \text{and} \quad y' = \int_0^s \sin\theta\, ds$$

with θ from Eq. (3′) and two-angle trigonometric identity and differential *ds* as used previously.