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Prediction of regime transition in three-phase sparged reactors using linear stability analysis

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HIGHLIGHTS

- Model for homogenous to heterogeneous regime transition in three phase systems.
- Model is based on linear stability analysis.
- Predicts critical gas holdup for slurry bubble columns and three-phase fluidization.
- Good agreement with published experimental results.

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ABSTRACT

The estimation of critical gas holdup at which the transition from homogeneous regime to heterogeneous regime occurs is crucial for the design and scale-up of multiphase reactors. A number of experimental and empirical studies are published in the literature, however, there exists a lack of modeling studies which can satisfactorily predict the flow regime transition in three-phase sparged reactors.

In the present work, the theory of linear stability analysis has been extended to investigate the hydrodynamic stability of three-phase sparged reactors (slurry bubble columns and three-phase fluidization). A mathematical model has been developed for the prediction of regime transition over a wide range of bubble size ($0.7–20 \times 10^{-3}$ m) and terminal rise velocity ($80–340 \times 10^{-3}$ m/s), particle settling velocity ($1–1000 \times 10^{-3}$ m/s), particle concentration (0.0007–30 vol%) and slurry density (800–5000 kg/m^3). It was observed that the developed model predicts the transition gas holdup within an absolute deviation of 12% for three-phase sparged reactors. It was also observed that the developed generalized stability criterion predicts the regime transition in two-phase systems satisfactorily when applied to bubble columns.

1. Introduction

Three-phase reactors typically involve gas sparging into a liquid containing suspended solid particles. For non-mechanically agitated systems three-phase reactors can be generally classified as either slurry or fluidized columns depending on the liquid and solid inlet and outlet conditions. The different operating conditions are schematically represented in Fig. 1 for: (A) Continuous slurry bubble column, where two-phase (liquid–solid) slurry is continuously passed through the sparged column. (B) Batchwise slurry bubble column, where there is no liquid and solid feed and the fixed volume of two-phase (liquid–solid) slurry in the column is continuously sparged. (C) Fluidized column, where the liquid stream passes through the sparged column containing a fixed mass of solid particles. For all three column configurations the power input from the gas sparging provides turbulent dispersion which assists in suspension of the solid particles. For both (A) continuous slurry and (C) fluidized column configurations the liquid throughput provides additional upward motion that assists in suspension of the solids. Continuous slurry bubble columns typically operate with relatively small particles, with unhindered settling velocities less than 10×10^{-3} m/s, to ensure that the particles are elutriated from the system. Conversely, for fluidized columns it is not desirable to have the solids elutriated from the suspension, so (larger) particles with settling velocities greater than the superficial liquid velocity are used.

Three-phase reactors operate in either the homogeneous (bubbly) or heterogeneous (churn-turbulent) flow regimes. The homogeneous regime is characterized by uniform bubble and solid

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Nomenclature

A	parameter of the stability criterion, dimensionless
B	parameter of the stability criterion, m/s
c	solid content in slurry, vol%
C	parameter of the stability criterion, m^2/s^2
C_D	drag coefficient, dimensionless
C'_D	dimensional drag coefficient, kg/m^3 s
C_V	virtual mass coefficient, dimensionless
C_{V0}	virtual mass coefficient for isolated bubble, dimensionless
d_B	bubble diameter, m
d_P	particle diameter, m
D	column diameter, m
D_G	gas phase dispersion coefficient, m^2/s
D_i	internal diameter, m
D_0	outer diameter, m
D_{SL}	slurry phase dispersion coefficient, m^2/s
DR	diameter ratio of the annular gap column, dimensionless
E	parameter of the stability criterion, m^2/s
F	parameter of the stability criterion, 1/s
$f(\epsilon_G)$	function defined in Eq. (58), dimensionless
f_1	parameter appearing in Eq. (43), dimensionless
f_D	drag force on a single bubble, N
f_G	gravitational force on a single bubble, N
f_B	buoyancy force on a single bubble, N
F_z	z component of interaction force per unit volume, N/m^3
F_D	total drag force per unit volume, N/m^3
F_{VM}	virtual mass force per unit volume, N/m^3
g	acceleration due to gravity, m/s^2
$g(\epsilon_G)$	function defined in Eq. (57), dimensionless
g_z	z component of acceleration due to gravity, m/s^2
G	parameter of the stability criterion, m/s^2
h	bed height, m
H	parameter of the stability criterion, m^4/s^2
$h(r^*_B)$	function defined in Eq. (60), dimensionless
I	parameter of the stability criterion, m^3/s^2
k	wave number defined in Eq. (42), 1/m
K_1	constant in Eq. (31), dimensionless
K_2	constant in Eq. (35), dimensionless
K_3	constant in Eq. (39), dimensionless
l	integral length scale of turbulence, m
m	Richardson–Zaki index, dimensionless
Mo	Morton number, $g\mu_{SL}^4\Delta\rho/(\rho^2\sigma^3)$, dimensionless
$\underline{P}$	instantaneous pressure, N/m^2
P	average pressure, N/m^2
P'	fluctuating pressure, N/m^2
r_B	bubble radius, m
r^*_B	dimensionless bubble radius, dimensionless
Re_B	bubble Reynolds number, $d_B V_{B\infty}\rho_{SL}/\mu_{SL}$, dimensionless
s	growth rate defined by Eq. (42), 1/s
t	time, s
Ta	Tadaki number, $\mathrm{Re}_B Mo^{0.23}$, dimensionless
u	integral velocity scale of turbulence, m/s
u_0	steady state slurry velocity, m/s
u_1	perturbation in slurry velocity, m/s
u'_{SL}	slurry phase rms turbulent velocity, m/s
u_z	z component of slurry velocity, m/s
u'_z	z component of fluctuating slurry velocity, m/s
$(\underline{u}'_z)_{inlet}$	z component of fluctuating slurry velocity at inlet, m/s
u_z	z component of average slurry velocity, m/s
U_{SL}	superficial slurry velocity, m/s
v_0	steady state gas velocity, m/s
v_1	perturbation in gas velocity, m/s
v_z	z component of gas velocity, m/s
$\underline{v}'_z$	z component of fluctuating gas velocity, m/s
v_z	z component of average gas velocity, m/s
$V_{B\infty}$	terminal bubble rise velocity, m/s
V_G	superficial gas velocity, m/s
V_{GC}	superficial gas velocity at transition, m/s
V'_G	gas phase drift flux, m/s
V_L	superficial liquid velocity, m/s
V_P	particle/solid velocity, m/s
V_S	slip velocity, m/s
z	axial coordinate, m
Z	parameter of the stability criterion, m^2/s^2

Greek letters

α	proportionality constant for dispersion coefficient, dimensionless
Δt	change in time, s
ΔV_G	change in superficial gas velocity, m/s
ϵ_G	fractional gas holdup, dimensionless
ϵ_{GC}	fractional gas holdup at transition, dimensionless
$\underline{\in}'_G$	fluctuating fractional gas holdup, dimensionless
$\in_G$	average fractional gas holdup, dimensionless
ϵ_G	fractional gas holdup, dimensionless
ϵ_S	fractional solid holdup, dimensionless
ϵ_{SL}	fractional slurry holdup, dimensionless
ϵ_{SL0}	steady state fractional slurry holdup, dimensionless
$\epsilon_{\underline{S}L1}$	perturbation in fractional slurry holdup, dimensionless
$\in_{SL}$	average fractional slurry holdup, dimensionless
$\in'_{SL}$	fluctuating fractional slurry holdup, dimensionless
μ_G	gas viscosity, kg/ms
μ_M	mixture viscosity defined in Eq. (59), kg/ms
μ_{SL}	slurry viscosity, kg/ms
ν_t	turbulent kinematic viscosity, m^2/s
ρ_G	density of gas phase, kg/m^3
ρ_P	density of particles, kg/m^3
ρ_L	density of liquid phase, kg/m^3
ρ_{SL}	density of slurry phase, kg/m^3
$\Delta\rho$	difference between slurry and gas densities, kg/m^3
σ_{SL}	surface tension of slurry, N/m

Subscripts

0	initial steady state
1	perturbation
B	bubble
G	gas phase
L	liquid
S	solid
SL	slurry phase
∞	unhindered (in an infinite medium)

concentration and uniform bubble size throughout the system. There exists no liquid circulation as well as other phenomena such as coalescence and break-up. With an increase in the gas flow rate, transition to the heterogeneous regime is observed which is characterized by non-uniform radial holdup profiles for both phases and large bubble size distribution accompanied by bulk liquid circulation. The homogeneous and heterogeneous regimes for slurry bubble column reactors are shown schematically in Fig. 2A and B, respectively. In gas–liquid-solid fluidization, the two regimes are also known as particulate fluidization and aggregative fluidization. The heterogeneity in this case has been completely considered with respect to the gas phase behavior. The three-phase fluidized bed in the absence of gas becomes a two-phase solid–liquid fluidized bed. The two-phase fluidized bed itself can operate