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Description of the performances of a thermo-mechanical energy harvester using bimetallic beams

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ABSTRACT

Many recent researches have been focused on the development of thermal energy harvesters using thermo-mechanical or thermo-electrical coupling phenomena associated to a first-order thermodynamic transition. In the case of the bimetallic strip heat engine, the exploitation of the thermo-mechanical instability of bimetallic membranes placed in a thermal gradient enables to convert heat into kinetic energy. This paper is a contribution to the modeling and the comprehension of these heat engines. By restraining the study to the simply-supported bimetallic beams and using a Ritz approximation of the beam shape, this paper aims to give an analytical solution to the first mode of the composite beams and then to evaluate the efficiency of the harvesters exploiting these kinds of instability.

1. Introduction

A new class of heat engines has recently emerged to propose an alternative to the conventional thermoelectric generators that exploits the Seebeck effect to harvest low-grade heat and to power Wireless Sensors Networks. These harvesters are mainly based on the use of a thermodynamic phase transition to directly transform heat into electric energy or, by two successive conversion steps, to transform heat into mechanical energy and then into electric energy with an electro-mechanical transducer. The use of the martensitic transition of ferromagnetic materials have been explored in [1,2] and the second-order transition of ferromagnetic to paramagnetic materials in [3]. The transformation of heat into mechanical energy using the martensitic transformation of Shape-Memory Alloy has been presented in [4]. Similarly, the fabrication of miniaturized heat engines based on the phase change of a fluid has been presented in [5,6]. In this paper we study the bimetallic strip heat engine that exploits the snap-through and the thermal hysteresis of a thermo-mechanically bistable membrane to convert heat into mechanical energy [7–10]. The working principle of this heat engine is presented in Fig. 1. When placed in a thermal gradient caused by two heat sources of different temperatures, the bistable membrane tends to oscillates and transports heat from a source to the other. Each time the membrane switches from a side to the other, it produces pulses of kinetic energy that can be harvested by an electromechanical transducer that can be either a piezoelectric [9,10] or an electret-based transducer [8,9]. Until now, main researches have addressed the fabrication of this harvester and on the demonstration of its capability to power Wireless Sensor Nodes [10]. However, in [11] we presented a first model explaining the occurrence of the thermal snap-through and we evaluated the performances of the heat engine. In this article, we propose a simplified model of the snap-through by using a Ritz approximation of the beam's shape and demonstrate that the bimetallic strip heat engine can be classified in the category of heat engine exploiting a first-order transition.

We finally explain how the thermo-mechanical properties of the bimetallic beams affect the performances of the heat engine by giving analytical expressions of the available mechanical energy and of the Carnot efficiency of the energy harvester.

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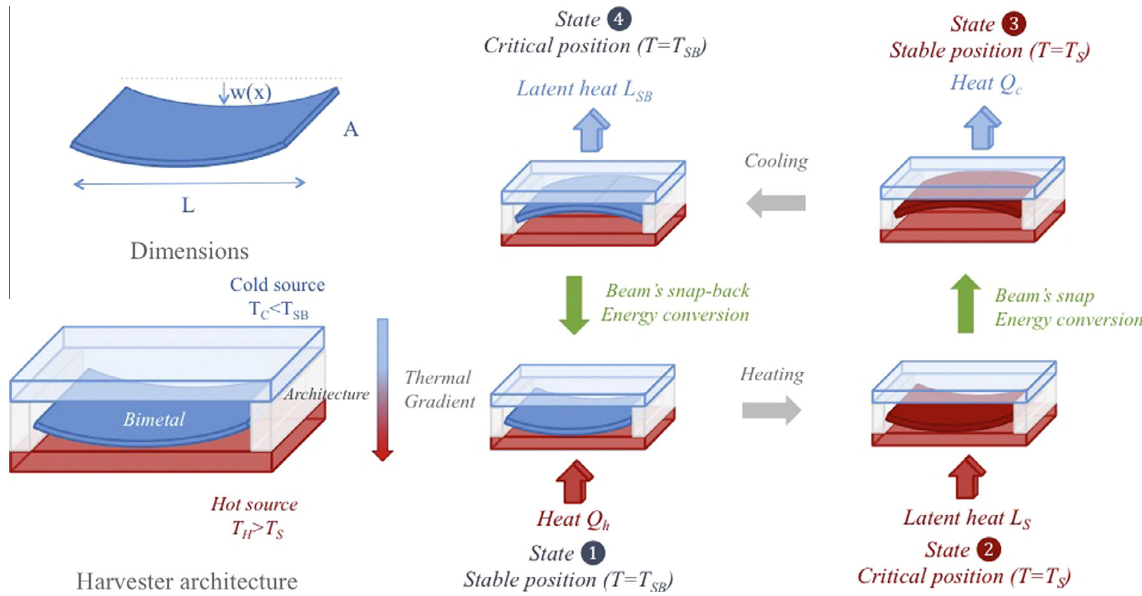

Fig. 1. Architecture and different phase of the operation of the bimetallic strip heat engine (T_S: snap temperature, T_{SB}: snap-back temperature, T_H: hot source's temperature, T_C: cold source's temperature).

2. Modeling of the thermal snap-through

2.1. Mechanical description of the thermal snap-through

The thermal snap-through of bimetallic beams (and more generally of composite beams) is a stability issue strongly linked to non-linearity of the Euler buckling phenomenon. The case treated in this article concerns the thermo-mechanical behavior of bimetallic beams that are submitted to the antagonistic effects of the residual stress and the asymmetry of the thermal expansion of the materials composing the beam. To simplify the study of the thermal snap-through, we take the case of a simply-supported beam (length L, thickness t, cross section A) made of two material layers having the same Young modulus E and same thickness $t/2$ (1). We introduce the difference of the coefficient of thermal expansion $\Delta\alpha$ (2), the mean residual stress $\bar{\sigma}$ and the residual stress difference $\Delta\sigma$ (3), and the mean thermal capacity (4).

$$E_1 = E_2 = \frac{E_1 + E_2}{2} = E; \quad t_1 = t_2 = t/2 \tag{1}$$

$$\Delta\alpha = \alpha_2 - \alpha_1 \tag{2}$$

$$\bar{\sigma} = \frac{\sigma_2 + \sigma_1}{2}; \quad \Delta\sigma = \sigma_2 - \sigma_1 \tag{3}$$

$$\overline{\rho \cdot c} = \frac{\rho_1 \cdot c_1 + \rho_2 \cdot c_2}{2} \tag{4}$$

Using these parameters, the law governing the behavior of the beams is (5). In this expression, the evolution of the displacement field w is for a part the consequence of the Euler equation and for the other part the consequence of the asymmetries of the residual stress and thermal stress. Because of the non-linearity of the Van-Karman strain tensor, the wave vector k is a function of the displacement w and the mean stress $\bar{\sigma}$ (6) [11].

$$k^2 \cdot w + \frac{\partial^2 w}{\partial x^2} = \frac{3}{2} \cdot \frac{\Delta\sigma/E - \Delta\alpha \cdot T}{t} \tag{5}$$

$$\frac{t^2 \cdot k^2}{12} = -\frac{\bar{\sigma}}{E} - \frac{1}{2 \cdot L} \cdot \int_0^L \left(\frac{\partial w}{\partial x}\right)^2 \cdot dx \tag{6}$$

In (6), the mean thermal expansion of the beam is neglected, which enables to find analytical solutions to the thermal snap-through by using the Rayleigh-Ritz method. By means of Eqs. (7), (5) and (6) can be simplified into a third-order polynomial (8).

The stability of the equilibrium positions is associated to the derivative of (8) that is a second-order polynomial. The thermal snap-through of the bimetallic beams is due to the loss of stability of the equilibrium ad thus associated to the roots of this polynomial (9).

$$w(x) = a \cdot \sin\left(\frac{\pi}{L} \cdot x\right) \tag{7}$$

$$\frac{3 \cdot \pi^2}{L^2} \cdot a^3 + \left(\pi^2 \cdot \frac{t^2}{L^2} + 12 \cdot \frac{\bar{\sigma}}{E}\right) \cdot a + \frac{6 \cdot t}{\pi} \cdot \left(\frac{\Delta\sigma}{E} - \Delta\alpha \cdot T\right) = 0 \tag{8}$$

$$\frac{9 \cdot \pi^2}{L^2} \cdot a^2 + \left(\pi^2 \cdot \frac{t^2}{L^2} + 12 \cdot \frac{\bar{\sigma}}{E}\right) = 0 \tag{9}$$

Eq. (9) shows that the thermo-mechanical bistability only appears if the mean residual stress exceeds the Euler buckling load σ_E (10). In this case, the beam losses its stability when the temperature verifies (11). These two temperatures, called snap temperature T_S and snap-back temperature T_{SB} characterize the hysteretic behavior of the bimetallic beam and are visible in Fig. 2. The value of the thermal hysteresis of the beam is given by (12).

$$-\bar{\sigma} \geqslant \frac{E \cdot t^2}{12} \cdot \frac{\pi^2}{L^2} = \sigma_E \tag{10}$$

$$T_S = \frac{\Delta\sigma}{E \cdot \Delta\alpha} + \frac{\Delta T}{2}; \quad T_{SB} = \frac{\Delta\sigma}{E \cdot \Delta\alpha} - \frac{\Delta T}{2}; \tag{11}$$

$$\Delta T = \frac{2 \cdot \pi^3}{27} \cdot \frac{t^2}{L^2 \cdot \Delta\alpha} \cdot \sqrt{-\left(1 + \frac{\bar{\sigma}}{\sigma_E}\right)^3} \tag{12}$$

The integration of (8) leads to find the fourth-order polynomial (13) representing the beam's strain energy. If the linear term depending on the temperature vanishes, the equation corresponds to the energy of a perfectly straight beam that buckles when the mean residual stress exceeds the Euler buckling load σ_E. In that case, the strain energy exhibits a double symmetric potential well separated by a potential barrier in $a = 0$ associated to a non-buckled shape, and the beam's equilibrium path is a pitchfork bifurcation.