Analysis of photonic spectra in Thue-Morse, double-period and Rudin-Shapiro quasiregular structures made of high temperature superconductors in visible range

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Abstract

The present paper attempts to determine the properties of photonic spectra of Thue-Morse, double-period and Rudin-Shapiro one-dimensional quasiperiodic multilayers. The supposed structures are constituted by high temperature HgBa\text{2}Ca\text{2}Cu\text{3}O\text{10} and YBa\text{2}Cu\text{3}O\text{7} superconductors. Our investigation is restricted to the visible wavelength domain. The results are demonstrated by the calculation of transmittance using transfer matrix method together with Gorter-Casimir two-fluid model. It is found that by manipulating the parameters such as incident angle, polarization, the thickness of each layer and operation temperature of superconductors the transmission spectra exhibit some interesting features. This paper, provides us a pathway to design tunable total reflector, optical filters and optical switching based on superconductor quasiregular photonic crystals.

1. Introduction

Photonic crystals (PCs) are periodically structured electromagnetic media, generally possessing photonic band gaps (PBGs). These PBGs represent ranges of frequency in which light cannot propagate through the structure, i.e., whether photons propagate through this structure or not depends on their wavelength. The simplest form of a PC is the one-dimensional (1D) periodic structure. It consists of a stack of alternating layers having low and high refractive indices. The layers thicknesses satisfy the Bragg condition [1]. In general, the dynamical control of electromagnetic wave transmission through a PC depends on the geometry and the refractive index of the dielectric materials [2,3].

In the past two decades, there has been an increasing interest in the study of quasi-regular (quasi-periodic) structures which have shown significant and interesting physical properties [4]. These structures behave much like disordered ones but which are constructed according to a deterministic procedure. They possess the properties of both periodic and random structures and have some distinct features not found in traditional media [5]. Experimental evidences toward understanding these new class of quasi-crystal were given by Schechtman et al. [6] and Levine and Steinhardt [7]. Photonic quasicrystals exhibit unique influence on the optical properties such as optical transmission and reflectivity, photoluminescence, light transport, plasmonics and laser action, etc. Li et al. [8] and Luo et al. [9] proposed two-dimensional photonic crystals that achieve multimode lasing action, low pumping threshold and excellent linear polarization property as well as wide directional dependence. This opens a new field of research in photonics in view of their vast technical applications. Photonic band gap properties of quasi-periodic multi-layered structures have been extensively studied for different materials [10]. Specifically, 1D photonic quasi-crystals are very important because their formation is relatively easy and they may provide the description of light propagation in one direction [11–13]. Within the group of studied quasiregular structures one find the so-called Fibonacci sequence (FS), Thue-Morse sequence (T-M), double-period (DP) sequence and Rudin-Shapiro (RS) sequence, etc [4]. There are several reports on the theoretical study of quasi-periodic structures arranged according to the FS [14,15]. But, scarce reports regarding the study of T-M, DP and RS systems are found in the published works especially about superconductors.

Superconductivity is a phenomenon whereby a number of pure metals and alloys offer no resistance to the passage of electrical current below a certain critical temperature. If a current is set up in a closed loop of such a material, in principle it will continue to flow forever. This means it is possible to pass electrical power through...
such a material without losing any energy in the process. Superconductivity was first discovered in mercury in 1911, and was rapidly followed by other materials such as lead and niobium. The critical temperature is a characteristic of the material, and for many years after the discovery of the first superconductor had thresholds around a few degrees Kelvin. This meant that in order to observe the effect, the material had to be cooled using liquid helium, which is both extremely costly and difficult to handle. The story changed dramatically in 1986, when J. Georg Bednorz produced a material, \textit{La}_2\textit{CuO}_4, with a threshold temperature of 35 K [16]. This material is different from the previous classes of material in that it has a complex crystal structure made from several components, based around copper oxide units. This discovery was followed a year later by Paul Chu and colleagues, of another material, \textit{YBa}_2\textit{Cu}_3\textit{O}_7 (often abbreviated to \textit{YBCO}) which had an even higher threshold temperature of around 93 K. This meant for the first time that a material exhibited superconducting behavior at temperatures above that of liquid nitrogen (77 K), which is much cheaper and easier to handle than liquid He. Many possible applications of this and related high temperature superconducting materials have been discussed. For example, ultra-fast microelectronics or instrumentation based on Josephson Effect, magnetically-levitated train, MRI (magnetic resonance image) and nuclear magnetic resonance (NMR) [17]. The structural, electrical, and magnetic properties of these materials have been well understood in the previous years. In this paper, based on the transfer matrix method (TMM) and two-fluid model [17], we intend to investigate the effect of the different parameters on photonic transmission spectra of three different quasiperiodic structures as Thue-Morse (T-M), double-period (DP) and Rudin-Shapiro (RS) containing high temperature superconductor layers in the visible region. There is twofold purpose of adopting the superconductor. First, the ability of extending PBG in the all-dielectric PCs is limited and the metal-dielectric PCs is inevitable to face the inherent loss issue. Such loss issue can be remedied by utilization of superconductor instead. Second, the dielectric function of the superconductor is depending on the external temperature, so it is possible to design thermally-tunable reflectors.

2. Supposed quasiperiodic structures and method of calculation

The quasiperiodic structures considered in this work are of the type generally known as substitutional sequences. The sequences generated by substitutions have been studied in several areas of mathematics, computer science, and cryptography [18]. The more recent applications in physics have been outlined in the Introduction. The sequences are characterized by the nature of their Fourier spectrum, which can be dense pure point (as for Fibonacci sequences) or singular continuous (as for T-M and DP and RS sequences).

2.1. Thue-Morse (T-M)

The T-M sequence first came about as the result of systematic studies of aperiodic chains initiated by Thue in 1906. Its results have been rediscovered many times since then, but the most important contribution to the sequence was made in 1921 by Morse in the context of topological dynamics [4]. Although there are several ways to define the T-M sequence, it is easy to prove they are equivalent to each other. In its simplest way, the T-M sequence can be defined by the recursive relations \( S_n = (S_{n-1}S_{n-1}) \) and \( S_n^0 = (S_{n-1}^0S_{n-1}^0) \) (for \( n \geq 1 \)), with \( S_0 = A \) and \( S_0^0 = B \). Another way to build up this sequence is through the inflation rules \( A \rightarrow AB \) and \( B \rightarrow BA \).

\[
S_0 = A; \quad S_1 = AB; \quad S_2 = ABBA; \quad S_3 = ABBABAAB; \quad \text{etc.} \quad (1)
\]

The number of building blocks in this quasiperiodic system increases with \( n \) as \( 2^n \), while the ratio of the number of the building blocks \( A \) to the number of the building block \( B \) is constant and equal to unity.

2.2. Double-period (DP)

The double-period sequence, is one of the newest of aperiodic chains. It has its origin in the study of system dynamics and in laser applications to nonlinear optical fibers [4]. Its recursion relation is somewhat similar to the T-M sequence case: the \( n \)th stage is given \( S_n = (S_{n-1}^0S_{n-1}) \) and \( S_n^0 = (S_{n-1}S_{n-1}) \) (for \( n \geq 1 \)), with \( S_0 = A \) and \( S_0^0 = B \). It is also invariant under the transformations \( A \rightarrow AB \) and \( B \rightarrow AA \). The first generations are

\[
S_0 = A; \quad S_1 = AB; \quad S_2 = ABAA; \quad S_3 = ABAABAB; \quad \text{etc.} \quad (2)
\]

The number of building blocks for DP sequence increases with \( n \) similar to the T-M sequence, i.e., like \( 2^n \). However, the ratio between the number of the building blocks \( A \) to the number of the building blocks \( B \) is not constant: it tends to 2 as the number of the generations goes to infinity.

2.3. Rudin-Shapiro (RS)

In mathematics the RudinShapiro sequence, also known as the GolayRudinShapiro sequence is an infinite automatic sequence named after Marcel Golay, Walter Rudin and Harold S. Shapiro, who independently investigated its properties. The quasiperiodic RS structure belongs to the family of substitutional sequences and displays an absolutely continuous Fourier spectrum, a property which it shares with the random structures. The RudinShapiro sequence is interesting because it does not satisfy the conditions for several theorems that apply to many quasisquare sequences, and thus exhibits differences in the spectrum properties, as compared to other sequences like the Fibonacci one. It also has a growing law for the number of constituent materials that increases more quickly than other sequences like Fibonacci one [4]. The Rudin-Shapiro arrays can be generated by the two-letter inflation rule as follows: \( AA \rightarrow AAAB, AB \rightarrow AABA, BA \rightarrow BBAB \) and \( BB \rightarrow BBBA \). The first generations are

\[
S_0 = A; \quad S_1 = AA; \quad S_2 = AABA; \quad S_3 = AAABAB; \quad \text{etc.} \quad (3)
\]

Similar to the previous mentioned sequences, again the number of building blocks for this sequence with \( n \) is \( 2^n \).

2.4. Transfer matrix method (TMM)

In this paper, we take a certain level of 1D T-M, DP and RS quasiperiodic structures to calculate the transmission spectra of these deterministic disorder multilayers. The transmission spectra of a layered system can be calculated by using transfer matrix method. For this purpose, we assume that a wave be incident from air with angle \( \theta \) onto the supposed multilayer structure. For the transverse electric (TE) wave, the electric field \( E \) is assumed in the \( x \) direction (the dielectric layers are in the \( xy \) plane), and the \( z \) direction is normal to the interface of each layer. When such an electromagnetic wave propagates through this multilayer structure, the incident, reflected and transmitted electric fields are connected via a transfer matrix \( M \) [1] as