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Dissipation in a quantum-mechanical system

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Abstract

We consider a model of a dissipative quantum-mechanical system consisting of weakly coupled quantum and classical subsystems. The classical subsystem is assumed to be infinite, and thus serves as a means to transfer the energy of the quantum subsystem to the infinity (actually, to dissipate the energy). The quantum-classical coupling is treated in the spirit of the *mean-field* approximation. Solving the equations for the classical subsystem explicitly an effective *dissipative* Schrödinger equation for the quantum subsystem is obtained. The proposed method is illustrated by calculating the shape of the nonlinear resonance.

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1. Introduction

For a long time, the main task of the quantum-mechanical description of physical systems was the calculation of the energy levels (the eigenvalues of the stationary Schrödinger equation) and the corresponding eigenfunctions as the main features of the system. The optical and transport properties of solids were mainly interpreted in terms of quantum transitions between these stationary states. The transitions were treated as momentary and calculated perturbatively by means of the Fermi's golden rule. The investigation of the dynamics involved in a quantum transition was started in the study of nuclear and electronic magnetic resonances, and later, of resonant transitions in laser amplifiers (see, for instance [1]). Recent experiments with femtosecond light pulses have initiated the study of quasiparticle dynamics in many-body systems of solids [2]. A pivotal role in quantum mechanical dynamic processes is played by dissipation as a way to balance the energy influx to the system from the external fields. The standard tool to describe dissipation in a quantum system is the density-matrix equation [3] which is natural for an *open* system, whereas the less arduous Schrödinger equation is reckoned to be appropriate for an *isolated* system, mathematically expressed by the hermiticity of the Hamiltonian.

With the present work, we emphasize that quantum dissipation can be captured within the Schödinger-equation formalism if the quantum-mechanical system under consideration is coupled to a classical counterpart that includes dissipative terms or is infinite, thus ensuring the transfer of the energy away from the quantum subsystem. We consider the interaction of a quantum oscillator with an infinite classical chain in the mean-field approximation and, explicitly solving the classical equations of the chain, derive a dissipative Schrödinger-like equation for the quantum system. The application of the obtained equation is illustrated by considering the nonlinear resonance.

2. Interaction of a quantum oscillator with a classical chain

We consider a quantum oscillator (described by the operators $\hat{x}$ and $\hat{p}$) coupled to the end of a semi-infinite classical string modeled by a chain of balls interconnected by springs. The system is presented in Fig. 1 and described by the Hamiltonian

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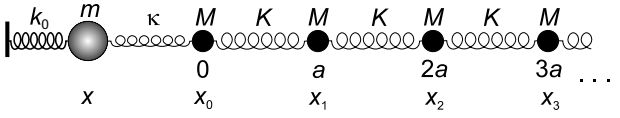

Fig. 1. The layout of the system.

$$H=\frac{1}{2m}\hat{p}^2+\frac{k_0}{2}\hat{x}^2+\frac{\kappa}{2}(\hat{x}-x_0)^2 + \sum_{n=0}^{\infty}\left\{\frac{1}{2M}p_n^2+\frac{K}{2}(x_n-x_{n+1})^2\right\}. \tag{1}$$

Following [4], we assume that the quantum oscillator is described by the wave function $\Psi(x,t)$ which solves the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(x,t)=H\Psi(x,t), \tag{2}$$

while the coordinates of the balls in the chain obey the classical Newton equations

$$\frac{\mathrm{d}}{\mathrm{d}t}x_n=\frac{\partial}{\partial p_n}\langle H\rangle,\quad \frac{\mathrm{d}}{\mathrm{d}t}p_n=-\frac{\partial}{\partial x_n}\langle H\rangle \tag{3}$$

with the averaged Hamiltonian

$$\langle H\rangle=\int_{-\infty}^{\infty}\mathrm{d}x\Psi^*(x,t)H\Psi(x,t). \tag{4}$$

This averaging constitutes the main assumption of the quasi-classical approximation.

The standard classical equations for the chain

$$M\frac{\mathrm{d}^2}{\mathrm{d}t^2}x_0+K(x_0-x_1)+\kappa x_0=\kappa\langle x\rangle, \tag{5a}$$

$$M\frac{\mathrm{d}^2}{\mathrm{d}t^2}x_n-K(x_{n-1}-2x_n+x_{n+1})=0,\quad n\geqslant 1 \tag{5b}$$

are solved by the Fourier transformation

$$x_n=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}k\mathrm{e}^{\mathrm{i}(kan-\omega t)}f(k), \tag{6}$$

with $\omega=vk$ and $v=a\sqrt{K/M}$ in a long-wave approximation ($ka\ll 1$). Then it follows from the boundary condition (5a) that:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}k\mathrm{e}^{-\mathrm{i}\omega t}\{K(1-\mathrm{e}^{\mathrm{i}ka})-Mv^2k^2+\kappa\}f(k) \approx\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}k\mathrm{e}^{-\mathrm{i}\omega t}\{-\mathrm{i}Kka+\kappa\}f(k)=\frac{Ka}{v}\frac{\mathrm{d}x_0}{\mathrm{d}t}+\kappa x_0=\kappa\langle x\rangle \tag{7}$$

The obtained equation of motion for the first ball (x_0) together with the Schrödinger equation. (2) constitute the complete set of equations that describe the behavior of the dissipative quantum system. Introducing new dimensionless coordinates by the transformation

$$\hat{x}\to l\hat{x},\quad \hat{p}\to\frac{\hbar}{l}\hat{p},\quad x_0\to l_0x_0,\quad t\to\omega_0^{-1}t, \omega_0=\sqrt{\frac{k_0}{m}},\quad \Omega=\sqrt{\frac{K}{M}},\quad l^2=\frac{\hbar}{m\omega_0},\quad l_0^2=\frac{\hbar}{M\Omega}, \tag{8}$$

we rewrite the above set of equations as follows

$$\mathrm{i}\frac{\partial}{\partial t}\Psi(x,t)=H\Psi(x,t), \tag{9a}$$

$$H=\frac{1}{2}\hat{p}^2+\frac{1}{2}\hat{x}^2-\lambda\hat{x}x_0, \tag{9b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}x_0+\lambda\left(\frac{l_0}{l}\right)x_0=\lambda\langle x\rangle, \tag{9c}$$

where

$$\lambda=\frac{\kappa}{k_0}\sqrt{\frac{\omega_0 m}{\Omega M}}. \tag{10}$$

Note that we excluded terms which do not depend on the operators $\hat{x}$ and $\hat{p}$ from (9b) as it can always be done by the proper transformation of the wave-function phase. The obtained equations describe the interaction of a quantum-mechanical system with a single dissipative classical mode x_0. They are general and can be applied to any quantum system with dissipation. For instance, in the case of a nonlinear (anharmonic) oscillator subject to an external time-periodic force we supplement the above Hamiltonian with the following additional terms:

$$\Delta H=-\hat{x}f\cos(\omega t)+\alpha\hat{x}^4. \tag{11}$$

The dimensionless coupling constant (10) and the ratio l_0/l depends on the parameter m/M. Thus, in the adiabatic case ($m\ll M$) the coupling constant is small, and this smallness ensures adequacy of the presented quasiclassical consideration.

3. Illustration

As an illustration, we consider the resonant absorption. Close to the resonant frequency of the linear oscillator ($\omega=1+\eta$, $|\eta|\ll 1$) it is convenient to expand the wave function into the series of harmonic oscillator eigenfunctions

$$\Psi(x,t)=\sum_{n=0}^{\infty}a_n(t)\mathrm{e}^{-\mathrm{i}\omega E_n t}\psi_n(x), \tag{12}$$

where $E_n=n+1/2$, and the functions ψ_n can be expressed in the standard way via Hermite polynomials. Inserting (12) into Eq. (9), assuming slow time variation of the coefficients a_n, and neglecting all oscillating terms (rotating wave approximation) we arrive at the following set of equations:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}a_n=-\{\eta E_n-\alpha\langle n|x^4|n\rangle\}a_n - \sqrt{n}(F+\mathrm{i}\gamma a^*)a_{n-1}-\sqrt{n+1}(F-\mathrm{i}\gamma a)a_{n+1}, \tag{13a}$$

$$a=\sum_{n=0}^{\infty}a_n a_{n+1}^*\sqrt{n+1},\quad F=f/2\sqrt{2},\quad \gamma=\lambda^2/2. \tag{13b}$$