New interpretation of the Haasen plot for solute-strengthened alloys

W.A. Curtin

School of Engineering, Brown University, Providence, RI 02912, USA

The Haasen plot (inverse activation area $1/\Delta a$ vs. offset flow stress $\sigma-\sigma_s$) for solute-strengthened alloys is usually assumed additive, $1/\Delta a = 1/\Delta a_s + 1/\Delta a_f$, with $1/\Delta a_f \sim \beta(\sigma-\sigma_s)$ due to forest interactions. Experiments often show a slope $<\beta$. Here, a model for the dislocation activation enthalpy is proposed that predicts a slope $1/(\Delta a_s \sigma_s)$ determined only by solute parameters Δa_s and σ_s and not directly connected to forest hardening. This parameter-free prediction agrees well with a wide range of experiments on Al–X alloys at $T = 78$ K.

The plastic behavior of metal alloys is determined by dislocation glide through a field of dispersed precipitates, forest dislocations and/or solute atoms introduced during alloying processing. The existence of several types of obstacles operating together is common in engineered alloys. To distinguish the individual mechanisms, their underlying material dependencies and their interactions, strain-rate sensitivity (SRS) measurements have been widely used [1,2] because strain-rate variations with stress are a sensitive measure of the factors controlling the thermally activated rate-dependence of dislocation motion and associated macroscopic plastic flow. The SRS is defined as $m = \frac{\partial ln\dot\varepsilon}{\partial ln\sigma}$, and an important related factor is the apparent activation area,

$$\Delta a = \frac{kT}{b\sigma}\frac{\partial ln\dot\varepsilon}{\partial ln\sigma} = \frac{kT}{b}\frac{\partial ln\dot\varepsilon}{\partial \sigma} \qquad (1)$$

The strain rate is usually assumed to arise from thermally activated dislocation motion, $\dot\varepsilon = \dot\varepsilon_o e^{-\Delta G(\sigma)/kT}$, where $\Delta G(\sigma)$ is the activation enthalpy written as a function of the uniaxial load σ but related to the resolved shear stress through a Taylor factor M, $\sigma = M\tau$. From Eq. (1), the apparent activation area is then

$$\Delta a = \frac{1}{b}\frac{\partial(-\Delta G)}{\partial\sigma} \qquad (2)$$

When forest dislocations are the only strengthening mechanism, the strengthening scales as $\hat\sigma \sim 1/\ell$, where ℓ is the average spacing between forest dislocations. At fixed temperature and strain rate, the activation area is

proportional to ℓ, $\Delta a \sim \ell$. The "Haasen plot" of inverse apparent activation area $1/\Delta a$ vs. stress $\sigma(\dot\varepsilon, T)$ is then a straight line of some slope β passing through the origin, $\frac{1}{\Delta a} = \beta\sigma(\dot\varepsilon, T)$, demonstrating the Cottrell–Stokes relation [3,4]. The analysis of rate-dependent flow in metals has a long [3–7] and illustrious history that cannot be fully reviewed here.

When another strengthening mechanism operates in tandem with forest dislocations, the traditional analysis proceeds using additional assumptions. First, we denote the forest mechanism using a subscript "f" and the second mechanism using a subscript "s", nominally representing solute strengthening. The assumption is then that the temperature- and strain-rate-dependent flow stresses $\sigma_f(\dot\varepsilon, T)$ and $\sigma_s(\dot\varepsilon, T)$ of the two mechanisms are additive so that the flow stress is $\sigma(\dot\varepsilon, T) = \sigma_f(\dot\varepsilon, T) + \sigma_s(\dot\varepsilon, T)$. From the inverse of Eq. (1), the inverse activation areas are then also simply additive and, together with the CS relation for the forest strengthening mechanism, give the prediction

$$\frac{1}{\Delta a} = \frac{1}{\Delta a_f} + \frac{1}{\Delta a_s} = \frac{1}{\Delta a_s} + \beta(\sigma(\dot\varepsilon, T) - \sigma_s(\dot\varepsilon, T))$$
$$\sigma(\dot\varepsilon, T) \geqslant \sigma_s(\dot\varepsilon, T) \qquad (3)$$

where the slope β has the same value as when forests operate alone in the pure alloy. Experiments on various solute-strengthened alloys show that the Haasen plot is indeed linear at stresses larger than $\sigma_s(\dot\varepsilon, T)$ but with a slope that is clearly different, and usually lower, than the slope measured in the pure alloy (e.g. [1,8,9]); Figure 1 shows examples for some Al alloys along with the data for pure Al [1,9].
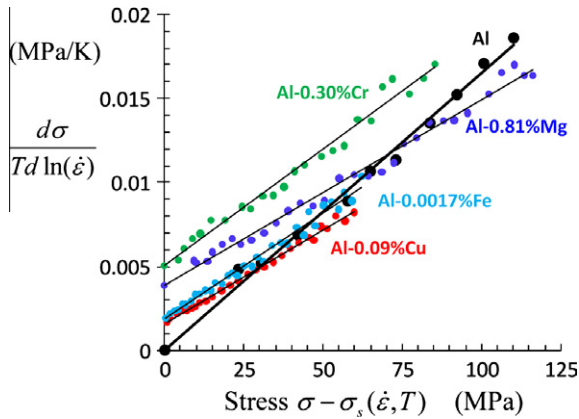
E-mail: william_curtin@brown.edu

Figure 1. Scaled inverse activation area, $k/b\Delta a$, vs. offset flow stress, i.e. the Haasen plot, for pure Al and for several solute-strengthened Al alloys [9,16].

The discrepancy between theory and experiment directs us to reconsider the assumption in the prior analyses. Additivity of the zero temperature flow stresses has been shown via computer simulations of point-pinning obstacles [10–12], valid when the length scale associated with mechanism "*s*" is much different from that for mechanism "*f*" (for solutes and precipitates, $\ell_s \ll \ell_f$) [12], and is supported by conceptual arguments [2]. However, additivity of the temperature- and strain-rate-dependent flow stresses and additivity of the inverse activation areas are both more speculative. Kocks et al. [2] justify these assumptions by extending the arguments used to justify additivity of the zero-temperature flow stress. In the special case of an enthalpy that depends linearly on stress (i.e. $\Delta G = \Delta G^o - \tau b\Delta a$), additivity of $\sigma_f(\dot{\varepsilon}, T)$ and $\sigma_f(\dot{\varepsilon}, T)$ and of the inverse activation areas does follow from additivity of the zero-temperature flow stresses because $\hat{\sigma} = \Delta G^o / b\Delta a$. However, in general, the assumed additivity remains uncertain. Simulations of thermally activated flow in the presence of multiple obstacles have been performed [13,14], but usually in a regime where the zero-temperature strengths are not additive. Those simulations show that thermally activated flow can be a complicated

combination of the multiple mechanisms, and do not confirm or refute the above assumptions. Monnet and Devincre [15] showed that forest strengthening can be reduced in the presence of a friction stress due to a reduction in the junction length. Although quantitative values suggest that such effects only arise for solute strengthening rather larger than the solute strengthening for the data shown in Figure 1 and Table 1, the Monnet–Devincre analysis shows that the solute–forest interaction is altered and this could translate into changes in the apparent forest activation area. However, the data in Figure 1 and Table 1 do not show any particular correlation between the Haasen slope and the solute flow stress which might be expected to emerge from the Monnet–Devincre analysis. Also, no direct connection to rate-dependent phenomena is made in this analysis. The deviation in Haasen slope between traditional theories and many experiments thus remains unexplained.

To rectify the discrepancy between experimental and theoretical slopes of the Haasen plot, we propose a different model for the rate-controlling process when multiple mechanisms are operating. When mechanisms "*f*" and "*s*" are operating simultaneously, $\Delta G(\sigma)$ is not known in general. Here, we use the well-justified additivity at zero temperature, $\hat{\sigma} = \hat{\sigma}_f + \hat{\sigma}_s$, and then postulate that if mechanism "*s*" is more easily thermally activated than mechanism "*f*" then the activation enthalpy is controlled $\Delta G_s(\sigma)$ but modified to include only the change in the zero temperature flow stress. Specifically, we replace $\sigma/\hat{\sigma}_s$ by $\sigma/(\hat{\sigma}_f + \hat{\sigma}_s)$, so that

$$\Delta G(\sigma) = \Delta G_s(\tilde{\sigma}); \tilde{\sigma} = \sigma\hat{\sigma}_s / (\hat{\sigma}_f + \hat{\sigma}_s) \quad (4)$$

To be concrete, within a standard general form for $\Delta G(\sigma)$ such as [2]

$$\Delta G(\sigma) = \Delta G^o \left(1 - \left(\frac{\sigma}{\hat{\sigma}} \right)^p \right)^q \quad (5)$$

the postulate is

$$\Delta G(\sigma) = \Delta G_s^o \left(1 - \left(\frac{\sigma}{\hat{\sigma}_f + \hat{\sigma}_s} \right)^{p_s} \right)^{q_s} \quad (6)$$

The corresponding activation area follows from Eq. (2) and the chain rule of differentiation as

Table 1. Experimental data on composition, flow stress, and Haasen plot intercept and slope [9,16], along with the model prediction (Eq. (10), ratio of (intercept)/(flow stress)) and the ratio between the predicted and measured values, for a wide range of solute-strengthened Al alloys.

	Experiments				Model	
	Solute atomic %	Flow stress (MPa)	Inverse activation volume intercept ×100 (MPa/K)	Haasen slope ×1000 (1/K)	Haasen slope ×1000 (1/K)	Haasen slope (predicted)/ (measured)
Al–Mg	0.4440	20.58	0.2860	0.0988	0.1390	1.41
Al–Mg	0.8102	34.18	0.3890	0.1100	0.1138	1.03
Al–Cu	0.0897	12.26	0.1300	0.1250	0.1060	0.85
Al–Cu	1.6504	86.57	0.9600	0.1310	0.1109	0.85
Al–Fe	0.0008	11.00	0.1100	0.0910	0.1000	1.10
Al–Fe	0.0017	16.79	0.2000	0.1172	0.1191	1.02
Al–Fe	0.0044	33.40	0.3420	0.0999	0.1024	1.02
Al–Cr	0.0026	9.66	0.0333	0.1320	0.0345	0.26
Al–Cr	0.0727	23.70	0.2850	0.1250	0.1203	0.96
Al–Cr	0.1040	11.25	0.3600	0.2090	0.3200	1.53
Al–Cr	0.3020	50.16	0.4830	0.1380	0.0963	0.70

Al–Cr(0.0727) is a single crystal.