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Scripta Materialia 54 (2006) 545–552
Scripta MATERIALIA
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Evaluation of elastic and thermoelastic properties of lotus-type porous metals via effective-mean-field theory

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Received 7 March 2005; received in revised form 20 October 2005; accepted 24 October 2005
Available online 28 November 2005

Abstract

The effective-mean-field (EMF) theory, consisting of Mori–Tanaka's mean-field theory and Bruggeman's effective medium approximation, was extended in order to calculate the coefficients of thermal expansion (CTE) of composite materials. The effective elastic constants and CTE of lotus-type porous metals, possessing cylindrical pores aligned unidirectionally, were evaluated with the EMF theory.

Keywords: Porous material; Elastic behaviour; Internal stresses; Mean field analysis; Effective-mean-field theory

1. Introduction

Most of the metals such as Cu, Mg and Fe possess the solubility gap of hydrogen at the melting point, i.e., hydrogen solubility discontinuously decreases with solidification. When such metals solidify under hydrogen atmosphere, the solubility gap yields the formation of numerous pores at the solid/liquid interfaces. For unidirectional solidification, the pores grow unidirectionally along the solidification direction. By utilizing this principle, Nakajima et al. have succeeded in fabricating lotus-type porous metals possessing cylindrical pores aligned unidirectionally (Fig. 1) [1–3]. Previous studies have revealed that the specific strength of lotus-type metals, in the direction parallel to the longitudinal axis of the pore, holds despite the presence of the pores [4,5]. Thus, the mechanical properties of lotus-type metals are superior to those of conventional porous metals possessing isotropic pores. Furthermore, the aligned long pores allow the permeability of fluid, and micron-size pores enlarge the surface area. These unique characteristics bring applications in various fields. When lotus-type metals are used as structural or high temperature components, the elastic constants and the coefficients of the thermal expansion (CTE) are needed to be clarified. The detailed understanding of them involves the clarification of the effects such as the pore shape and porosity.

Many researchers have proposed methods for calculating the elastic and thermoelastic properties of composite materials including porous materials [6–9]. Hashin and Shtrikman [10] have derived theoretical upper and lower bounds for the effective elastic constants, and the combination of Hashin–Shtrikman's (HS) bounds and Levin's relation [11] gives the bounds for the effective CTE [12]. However, these bounds cannot be derived analytically except in some special cases. Taya et al. [13,14] have proposed the method based on Eshelby's inclusion theory [15] and Mori–Tanaka's mean-field (MTMF) theory [16]. This method provides the effective elastic constants and CTE with the shape, alignment, and volume fraction of inclusions taken into account [17,18]. However, this method cannot give accurate values when the volume fraction of inclusions is high. This is because the far-field approximation in MTMF theory cannot sufficiently take account of elastic interaction among inclusions in the high-fraction region [19]. Recently, in order to overcome this problem, Tane and Ichitsubo [20] have proposed the effective-mean-field (EMF) theory by combining

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1359-6462/$ - see front matter
doi:10.1016/j.scriptamat.2005.10.049

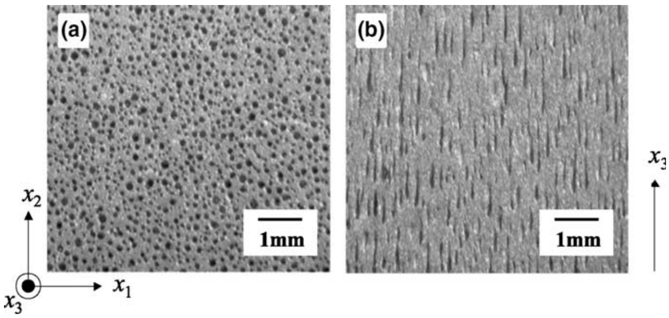

Fig. 1. Microstructures of (a) transverse section perpendicular to the longitudinal pore direction and (b) longitudinal section parallel to the pore direction for lotus-type porous copper, and the coordinate system of the specimen.

Bruggeman's effective medium approximation [21] and MTMF theory. This theory provides the effective elastic constants with high accuracy even when the inclusion fraction is high. However, the EMF theory has been applied only to the elastic problem until now.

In this paper, we extend the EMF theory to the thermoelastic problem in order to calculate the CTE of composite materials (including porous materials). First we introduce the EMF theory for the effective elastic constants of composite materials, and then newly derive the theory for calculating the effective CTE of composite materials on the basis of the EMF theory. Next, we examine the validity of the theory for the elastic constants and CTE by comparing the theory to theoretical upper and lower bounds. Finally, we evaluate the elastic constants and CTE of lotus-type porous metals on the basis of the EMF theory, and discuss their elastic and thermoelastic properties.

2. Effective-mean-field theory

2.1. Theory

2.1.1. Elastic constants

A composite material consists of a matrix and one type of inclusions, whose volume fraction is denoted by f_{M} and $f_{\mathrm{I}}(=1-f_{\mathrm{M}})$, respectively. The equations, $\bar{\boldsymbol{\sigma}}=f_{\mathrm{M}}\bar{\boldsymbol{\sigma}}_{\mathrm{M}}+f_{\mathrm{I}}\bar{\boldsymbol{\sigma}}_{\mathrm{I}}$ and $\bar{\boldsymbol{\epsilon}}=f_{\mathrm{M}}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}+f_{\mathrm{I}}\bar{\boldsymbol{\epsilon}}_{\mathrm{I}}$, express the spatial averages of the stress and strain in the composite material, where $\bar{\boldsymbol{\sigma}}_{\mathrm{M}}=\mathbf{C}_{\mathrm{M}}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}$ and $\bar{\boldsymbol{\sigma}}_{\mathrm{I}}=\mathbf{C}_{\mathrm{I}}\bar{\boldsymbol{\epsilon}}_{\mathrm{I}}$; $\mathbf{C}_{\mathrm{M}}$ and $\mathbf{C}_{\mathrm{I}}$ denote the elastic constants of the matrix and inclusion, respectively. (Bold face capitals represent 6×6 matrices, and bold face Greek characters represent 6×1 vectors.) Then, $\bar{\boldsymbol{\sigma}}=\overline{\mathbf{C}}\bar{\boldsymbol{\epsilon}}$ defines effective elastic constants of the composite material, $\overline{\mathbf{C}}$ [22]. The definition of $\mathbf{A}$ as $\bar{\boldsymbol{\epsilon}}_{\mathrm{I}}=\mathbf{A}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}$ provides the effective (macroscopic) elastic constants [23]:

$$\overline{\mathbf{C}}=(f_{\mathrm{M}}\mathbf{C}_{\mathrm{M}}+f_{\mathrm{I}}\mathbf{C}_{\mathrm{I}}\mathbf{A})(f_{\mathrm{M}}\mathbf{I}+f_{\mathrm{I}}\mathbf{A})^{-1}, \tag{1}$$

where $\mathbf{A}$ is the strain concentration factor and $\mathbf{I}$ is the unit matrix.

The combination of Eshelby's equivalent inclusion theory [15] and MTMF theory [16] gives $\mathbf{A}$ as follows. First, when a homogeneous substance suffers an external stress $\boldsymbol{\sigma}^{\mathrm{ext}}$, the stress field inside the substance is given by $\boldsymbol{\sigma}^{\mathrm{ext}}=\mathbf{C}_{\mathrm{M}}\boldsymbol{\epsilon}_{\mathrm{M}}$; $\mathbf{C}_{\mathrm{M}}$ denotes the elastic constants and $\boldsymbol{\epsilon}_{\mathrm{M}}$ denotes the (uniform) strain. Next, when an infinitesimal isolated inclusion is added to the substance, the internal stress inside the isolated inclusion with $\mathbf{C}_{\mathrm{I}}$ is given by

$$\boldsymbol{\sigma}_{\mathrm{I}}=\mathbf{C}_{\mathrm{I}}\boldsymbol{\epsilon}_{\mathrm{I}}=\mathbf{C}_{\mathrm{I}}(\boldsymbol{\epsilon}_{\mathrm{M}}+\boldsymbol{\gamma})=\boldsymbol{\sigma}^{\mathrm{ext}}+\boldsymbol{\sigma}^{\infty}, \tag{2}$$

where $\boldsymbol{\sigma}^{\infty}$ and $\boldsymbol{\gamma}$ represent extra internal stress and strain caused by the elastic inhomogeneity. When the inclusion is ellipsoidal, Eshelby's theory [15] provides $\boldsymbol{\sigma}_{\mathrm{I}}$ using the eigen strain of "an equivalent inclusion", $\boldsymbol{\epsilon}^{*}$:

$$\boldsymbol{\sigma}_{\mathrm{I}}=\boldsymbol{\sigma}^{\mathrm{ext}}+\boldsymbol{\sigma}^{\infty}=\mathbf{C}_{\mathrm{M}}(\boldsymbol{\epsilon}_{\mathrm{M}}+\boldsymbol{\gamma}-\boldsymbol{\epsilon}^{*}), \tag{3}$$

where the extra strain $\boldsymbol{\gamma}$ is given by $\boldsymbol{\gamma}=\mathbf{S}\boldsymbol{\epsilon}^{*}$. The terms $\mathbf{S}$ is Eshelby tensor depending on the matrix elastic constants and inclusion shape [15]; its matrix notation is given by Pedersen [24]. From Eqs. (2) and (3), the expression of $\boldsymbol{\sigma}^{\infty}$ can be obtained: $\boldsymbol{\sigma}^{\infty}=\mathbf{C}_{\mathrm{M}}(\boldsymbol{\gamma}-\mathbf{S}^{-1}\boldsymbol{\gamma})$. We next consider a composite material possessing finite concentration of inclusions. The MTMF theory [16] expresses the average internal stress of inclusions, using $\boldsymbol{\sigma}^{\infty}$ for an isolated inclusion, as

$$\bar{\boldsymbol{\sigma}}_{\mathrm{I}}=\bar{\boldsymbol{\sigma}}_{\mathrm{M}}+\boldsymbol{\sigma}^{\infty}, \tag{4}$$

which leads to the following equation:

$$\mathbf{C}_{\mathrm{I}}\bar{\boldsymbol{\epsilon}}_{\mathrm{I}}=\mathbf{C}_{\mathrm{M}}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}+\mathbf{C}_{\mathrm{M}}(\boldsymbol{\gamma}-\mathbf{S}^{-1}\boldsymbol{\gamma}). \tag{5}$$

Here, the expression of $\mathbf{A}$ as $\bar{\boldsymbol{\epsilon}}_{\mathrm{I}}=\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}+\boldsymbol{\gamma}=\mathbf{A}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}}$ and substitution of this equation into Eq. (5) yield $\mathbf{A}$:

$$\mathbf{A}=[\mathbf{S}\mathbf{C}_{\mathrm{M}}^{-1}(\mathbf{C}_{\mathrm{I}}-\mathbf{C}_{\mathrm{M}})+\mathbf{I}]^{-1}. \tag{6}$$

As shown in Eqs. (4) and (5), MTMF theory regards the nonuniform stress field in a matrix as the uniform one of the average stress, and derives the stress inside the inclusions by utilizing the solution for an isolated inclusion. When inclusions in composite materials are apart from one another, i.e., the volume fraction of inclusions is low, this approximation is valid [19]. However, the approximation cannot sufficiently take account of the elastic interaction among inclusions in the high-fraction region.

The EMF theory, consisting of MTMF theory and effective-medium approximation, deals with the elastic interaction as follows. The inclusions of low fraction Δf_{I} are added into a composite material suffering the external force $\boldsymbol{\sigma}_{\mathrm{ext}}$; the inclusion fraction of the pre-existence composite material is $n\Delta f_{\mathrm{I}}$. Then, the effective medium approximation takes account of the interaction between the added inclusions and the pre-existence inclusions. First, the composite material with inclusions of $n\Delta f_{\mathrm{I}}$ is regarded as the homogeneous matrix (effective medium) with the effective elastic constants of $\overline{\mathbf{C}}_{(n)}$ on the basis of the effective-medium approximation. Next, the low fraction of the added inclusions allows the MTMF theory to adequately express the average stress inside the added inclusions, $\boldsymbol{\sigma}_{\mathrm{I}(n)}$:

$$\mathbf{C}_{\mathrm{I}}\bar{\boldsymbol{\epsilon}}_{\mathrm{I}(n)}=\overline{\mathbf{C}}_{(n)}\bar{\boldsymbol{\epsilon}}_{\mathrm{M}(n)}+\boldsymbol{\sigma}^{\infty}_{(n)}, \tag{7}$$