

# Damage tolerance of an elastic-brittle diamond-celled honeycomb

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The fracture strength of an elastic-brittle, centrally cracked plate made from a diamond-celled lattice is calculated by finite element simulations. Conventional linear elastic fracture mechanics applies when the crack length much exceeds the cell size. But when the crack is only a few cell sizes, the stress concentration at the crack tip is negligible and the strength is comparable to the unnotched strength. An analytical model is derived for the fracture toughness.

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Ceramic honeycombs with the square cells shown in [Figure 1](#) are commonly used in catalytic converters and in particulate filters for automobiles. Ceramic foams and honeycombs also find application as filters for liquid metal due to their high chemical stability up to a high temperature. More recently, ceramic lattice materials and other types of brittle honeycombs have also been developed for bioengineering purposes, such as prosthetic scaffold implants. Tailoring the porosity of these structures allows for ingrowth of new bone tissue and also reduces problems associated with the mismatch of elastic properties. The brittle nature of ceramic honeycombs, together with the severe thermal shock and mechanical loads to which they are subjected, makes their damage tolerance a concern.

Early work on the fracture behaviour of brittle honeycombs made use of linear elastic fracture mechanics (LEFM) concepts to estimate the fracture toughness of a hexagonal honeycomb [1]. The stress field of an equivalent linear elastic continuum was used to calculate the stresses on the cell walls of the lattice directly ahead of the crack tip. The macroscopic fracture toughness was estimated by assuming that the critical strut directly ahead of the crack tip fails when the maximum tensile stress within it attains the fracture strength  $\sigma_f$ . These ideas were extended and refined by Huang and Gibson [2] for hexagonal and diamond-celled honeycombs to account for the statistical nature of the modulus of rupture  $\sigma_f$  of the cell-wall material. Huang and Gibson con-

cluded that the fracture toughness of the diamond-celled honeycomb scales as  $K_{IC} \propto (t/\ell)^2$ , where  $t$  is the thickness of each strut and  $\ell$  is its length. We shall reassess this result below for the diamond-celled honeycomb.

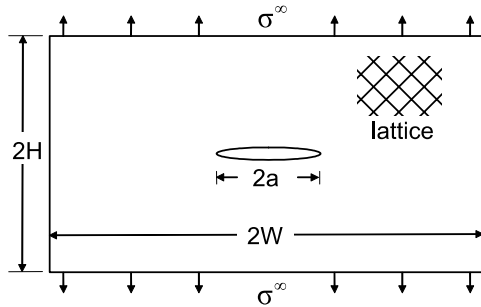
Recently, Fleck and Qiu [3] have made numerical and analytical predictions for the fracture toughness of several isotropic honeycombs: hexagonal, triangular and Kagome. They considered both semi-infinite cracks and finite cracks within a centre cracked panel. In the absence of a crack, the panel (with diamond-celled honeycomb microstructure) has an unnotched tensile strength  $\sigma_u$  which scales with the tensile strength of the solid  $\sigma_f$  and with the ratio  $\bar{t} = t/\ell$ , according to [4]:

$$\sigma_u = \frac{2}{3} \bar{t}^2 \sigma_f. \quad (1)$$

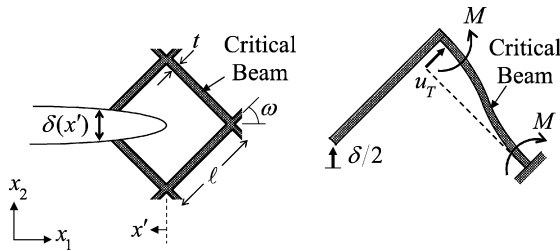
Now introduce a macroscopic crack into the honeycomb. At long crack lengths, the strength is dictated by the criterion of linear elastic fracture mechanics,  $K_I = K_{IC}$ , where  $K_I$  is the applied mode I stress intensity factor and  $K_{IC}$  is the mode I fracture toughness of the lattice material. Fleck and Qiu [3] showed that the transition in behaviour from *strength-control* to *toughness-control* occurs at the transition flaw size of  $a_T = K_{IC}^2 / \pi \sigma_u^2$ , in agreement with previous studies on fully dense specimens (see e.g. the review by Fleck et al. [5]).

In the present study, we explore the tensile fracture response of a centre-cracked plate (CCP) made from a diamond-celled honeycomb ([Fig. 1](#)). The plate contains a crack of length  $2a$  and the honeycomb is made from a solid of Young's modulus  $E_s$  and tensile fracture strength  $\sigma_f$ . The diamond-celled lattice (sketched in [Fig. 2](#)) is characterised by its cell size  $\ell$ , wall thickness

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**Figure 1.** Centre cracked plate (CCP) made from a diamond-celled lattice and subjected to uniaxial tension.



**Figure 2.** Sketch of the crack tip showing the idealised model of the deformation.

$t$  and core angle  $\omega$  (only square cells of  $\omega = 45^\circ$  are considered in this study). The cell wall material is assumed to be linear elastic up to fracture. The relative density of the diamond-celled lattice as defined by the density of the lattice divided by that of the solid from which the cell walls are made is given by

$$\bar{\rho} = \frac{t}{\ell} \left( 2 - \frac{t}{\ell} \right). \quad (2)$$

Throughout this study, the cell size  $\ell$  is held constant and we evaluate the sensitivity of the macroscopic strength to relative density, as parameterised by  $\bar{t} = t/\ell$ , and to the relative crack length  $a/\ell$ . The specimen width  $W$  and height  $H$  are taken to be sufficiently large in relation to the crack length for specimen size effects to be negligible. Consequently, the K-calibration for the homogeneous, orthotropic CCP can be taken as  $K = \sigma^\infty \sqrt{\pi a}$ .

The outline of the paper is as follows. An analytic estimate for the fracture toughness of the lattice  $K_{IC}$  is derived. The dependence of notched strength of the CCP is explored for a wide range of  $t/\ell$  and  $a/\ell$  by the finite element (FE) method. The notched strength is compared with the unnotched strength at short crack lengths, and with the LEFM prediction at long crack lengths. The role of the T-stress in influencing the fracture strength is also addressed.

Consider the cracked lattice shown in Figure 2. Assume that the bar immediately ahead of the crack tip is the critical bar of the lattice: it fails first. Further, assume that it deforms as a built-in beam, as sketched in Figure 2. The clamping moment  $M$  on this bar is

$$M = \frac{1}{2} E_s \frac{t^3}{\ell^2} u_T, \quad (3)$$

where the transverse displacement,  $u_T$ , is related to the crack tip opening displacement,  $\delta$ , evaluated at a distance  $x' = \ell/\sqrt{2}$  from the crack tip according to

$$u_T = \frac{\delta(x' = \ell/\sqrt{2})}{2\sqrt{2}}. \quad (4)$$

Recall that the crack tip opening displacement of an equivalent orthotropic continuum is given [6] by

$$\delta(x') = \frac{8}{\sqrt{2\pi}} CK_1 \sqrt{x'}, \quad (5)$$

where the elastic coefficient,  $C$ , for an orthotropic solid is given in Ref. [7]. For the orthotropic lattice under consideration,  $C$  is given by

$$C \simeq \frac{1}{\sqrt{2} E_s \bar{t}^2}. \quad (6)$$

Now assume that this critical beam fails when the local bending stress of  $\sigma_A = 6M/t^2$  attains the fracture strength of the cell wall material  $\sigma_f$ . Consequently, from Eqs. (3)–(7), we obtain

$$K_{IC} = \beta \sigma_f \bar{t} \sqrt{\ell}, \quad (7)$$

where the numerical constant  $\beta = 1/2$ . Numerical investigations [N.E. Romijn and N.A. Fleck, private communication] confirm the scaling of Eq. (11) with only a minor correction to the constant  $\beta$  of  $\beta = 0.44$ . Note that the linear dependence of  $K_{IC}$  upon  $\bar{t}$  contrasts with the quadratic dependence for hexagonal honeycombs [1,3], and with the dependence argued previously by Huang and Gibson [2] for a diamond-celled honeycomb.

A simple analytical estimate can be made for the transition flaw size  $a_T$ , at which the notched strength of the CCP switches from the unnotched value  $\sigma_u$  to the LEFM value

$$\sigma_c = K_{IC} / \sqrt{\pi a}. \quad (8)$$

The transition length follows from Eq. (7) as

$$\frac{a}{\ell} = \frac{9}{16\pi} \frac{1}{\bar{t}^2}. \quad (9)$$

Note that the transition length scales as  $\ell$ , but is very sensitive to  $\bar{t}$ . At small  $\bar{t}$ , such as 1%, we have a long transition crack length,  $a_T = 1790\ell$ .

Finite element simulations have been performed to determine the fracture strength  $\sigma_c$  of elastic-brittle CCP made from a diamond-celled honeycomb. The commercial finite element code ABAQUS (version 6.5-3) was used. Each strut in the lattice was modelled by a two-noded Euler–Bernoulli beam element (element type B23 in ABAQUS notation), which uses cubic interpolation functions and accounts for both stretching and bending deformation but neglects shear deformation. Two ways of introducing a crack into the lattice have been considered (Fig. 3). Crack morphology I has broken bars on each face of the crack (see Fig. 3a). Crack morphology II has intact bars but the joints are split on the crack plane (see Fig. 3b).

The net-section strength  $\sigma_c$  of the CCP is plotted against  $(\bar{t}^2 a/\ell)$  in Figure 4. This net strength has been normalised by the unnotched strength of the lattice

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