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Novel parameter-based flexure bearing design method

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ABSTRACT

A parameter study was carried out on the design variables of a flexure bearing to be used in a Stirling engine with a fixed axial displacement and a fixed outer diameter. A design method was developed in order to assist identification of the optimum bearing configuration. This was achieved through a parameter study of the bearing carried out with ANSYS®. The parameters varied were the number and the width of the arms, the thickness of the bearing, the eccentricity, the size of the starting and ending holes, and the turn angle of the spiral. Comparison was made between the different designs in terms of axial and radial stiffness, the natural frequency, and the maximum induced stresses. Moreover, the Finite Element Analysis (FEA) was compared to theoretical results for a given design. The results led to a graphical design method which assists the selection of flexure bearing geometrical parameters based on pre-determined geometric and material constraints.

1. Introduction

Flexure bearings are metal disks with spiral-like slots that enable them to flex in axial direction while exhibiting a much higher radial stiffness. They can be used to support shafts that perform a pure linear motion and are commonly used in free-piston Stirling machines in combination with linear motors. In these applications, their primary advantage is almost frictionless operation without requiring lubrication. They can be used in combination with clearance seals; they are inexpensive and can be easily manufactured by a dye in a punch press.

The concept of a flexure bearing or flexure spring was first introduced and patented by Wolf et al. in 1938 [1]. They used these bearings, mounted in a vibration detector, to capture the Earth's vibrations. Then, in 1981, flexure bearings were first used in a Stirling cryocooler at the University of Oxford [2]. Later, in 1992, Wong et al. [3] optimised a three-spiral flexure bearing using Finite Element Method (FEM) which was subsequently experimentally validated. These researchers also showed that the radial stiffness decreases with axial displacement and that the maximum stresses occur at the end of the spiral slot. In the same year, Marquardt et al. [4] proposed a design correlation for flexure bearings where the ratio of radial to axial spring stiffness was used to select the most suitable configuration for their application. The use of correlations in flexure bearing design was further developed by Wong et al. [5], who, in 1995, performed static and dynamic tests using FEM on a three-spiral slot flexure bearing. Their model results, verified through experimental dynamic testing, recommended that the dynamic stresses, rather than the static ones, be used for the fatigue analysis.

In 1996, Gaunekar et al. [6] analysed a three-spiral flexure bearing using FEA and found that the stresses increase with the axial displacement. Furthermore, they found that the axial and radial stiffness tend to have a linear behaviour when plotted against the axial displacement. They also noticed that the bearing's axial and radial stiffness also increases with increasing bearing material thickness. This work resulted in normalised graphs that assist in flexure bearing design.

Automated geometry creation for a three-spiral flexure bearing within Fortran was later achieved by varying the turn angle, thickness, and the outside radius. This work, undertaken by Lee and Pan [7], automated design generation and demonstrated that there were many design possibilities for a given low radial stiffness, but fewer for a high stiffness.

More recently, in 2007, Al-Otaibi and Jack [8] designed a flexure bearing for a linear-resonant motor with experimental validation of the FE results. Their findings suggested that an increase in turn angle of the spiral decreases the stresses and the axial stiffness, while the axial stiffness increases with the thickness of the disk. These findings were later supported by Simcock [9] who studied a flexure spring using FEM and performed experimental tests using strain gauges to measure bearing stresses. In 2012, Malpani et al. [10] and Kavade and Patil [11] performed a similar FEM study on

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Nomenclature

c_x	stress-raising factors	p	pitch
d	diameter of the starting and ending holes, m	r_0	radius at the beginning of the spiral, m
f_{nat}	natural frequency of the bearing, Hz	s	slot width, m
F_x	axial force, N	S_e	Endurance limit of flexure bearing, MPa
F_y	radial force, N	S'_e	Endurance limit of rotating beam specimen, MPa
ID	active inner active diameter, m	t	thickness of the disk, m
ID'	physical inner diameter, m	w	arm width, m
k_a	axial stiffness, N/m	δ_x	axial displacement, m
k_r	radial stiffness, N/m	δ_y	radial displacement, m
n	number of arms, m	Δx	displacement, mm
OD	active outer active diameter, m	Θ	turn angle, degrees
OD'	physical outer diameter, m	σ_{max}	axial maximum stress, Pa

a flexure bearing by varying the turn angle of the spiral and the thickness of the bearing. They also validated their results experimentally and found good agreement with their earlier work.

The purpose of this investigation is to test different parameters of a spiral flexure bearing design and analyse their influence in terms of fatigue when applied to a flexure bearing. The parameter study presented here indicates which parameters can be varied and what their impact is on bearing performance. In this study, parameters were varied that have not been analysed before, such as the size of the ending holes or the number of arms. This investigation also uses the stiffness ratio, originally introduced by Marquardt et al. [4] as a theoretical concept, to compare the different configurations under dynamic load. This ratio has not previously been used to compare several bearing designs obtained by using FEM. Furthermore, a modal analysis of the flexure bearing is also carried out using FEM. The dynamic stresses on the bearing are then analysed and minimised for a given design. Finally, as a result of the parameter study, a design method is developed in order to select the optimum geometric configuration of the bearing depending on pre-selected geometric and material design constraints. Large deflections effects for the displacements have been taken into consideration during the FE simulations in ANSYS®.

2. Geometry and definitions

This investigation considers the variation of geometric parameters, such as the thickness t of the disk, the number of arms n, the turn angle Θ, the diameter of the starting and ending holes d, and the slot width s that determines the arm width w, for a fixed active outer and inner diameter OD and ID, defined as the area that experiences flexure, a fixed stroke, and a fixed radius r_0 at the beginning of the spiral. The flexure bearing considered here had an outer diameter of OD' 90 mm, an active diameter OD of 80 mm, an active inner diameter ID of 20 mm, and a radius r_0 at the beginning of the spiral of 10 mm (Fig. 1). Also, a centre hole ID' (the inner diameter) with 8 mm diameter provided space for the shaft of the Stirling engine. Its design is based on the Archimedean spiral as described by Al-Otaibi and Jack [8]. The flexure bearing analysed has three arms ($n = 3$), the spirals make 1 turn ($\Theta = 360°$), the arm width w is set to 9.33 mm (for a 0.5 mm slot width s), and the thickness t of the disk is 0.7 mm. Also, the starting and ending holes of the spiral are set to a 1.5 mm diameter d.

3. Finite element analysis

3.1. Varying the turn angle and number of arms

The first parameters investigated in this paper were the turn angle of the spirals and the number of arms. The turn angle Θ was varied from 360° to 1080° in 90° increments, while the number of arms n were 2, 3, and 4. For this analysis the thickness of the bearing was 0.7 mm.

Once assembled in the engine, the bearing will oscillate at a specific frequency. In order to determine the permissible operating frequency, a modal analysis has to be carried out to find the natural frequency of the bearing in order to avoid the destructive consequences of resonance.

For this analysis, the displacement of the central area was held unconstrained in the axial direction, while the outer rim was fixed to meet the real boundary conditions. These conditions are shown in Fig. 2 where the shaded area around the centre hole is free to move in axial direction, while the shaded area at the outer rim is fixed.

These conditions were used for both the radial and axial tests. In the first case, a static bearing load was applied radially at the centre hole at zero stroke while a dynamic force was applied axially in the second test. In both cases the outer rim remained fixed. The material chosen for all the analyses was AISI 5160 which has the same characteristics as the materials used for metal springs (i.e. stainless steel with high yield strength), with a Young's modulus of 210 GPa, a Poisson's ratio of 0.29, and a density of 7850 kg/m^3.

Fig. 3 shows the mesh used for the analysis conducted with ANSYS 15. The meshing parameters were set to have a great number of elements at the starting holes of the spirals, where the stresses are expected to be concentrated.

Resonance causes unexpected motions on the bearing that can lead to premature failure if the operating frequency is near the natural frequency. In order to avoid this situation, an operating frequency below the natural frequency was selected for the subsequent analysis within this study. Comparison between the results of the modal analysis and those given by Wahl's correlation [12], shown in Fig. 4, shows that the natural frequency decreases with increasing turn angle and increasing number of arms, however, with a much stronger influence of the turn angle. It can be noticed that there is a good correlation between Wahl's correlations [12] and FE results.

Subsequently, a dynamic load was applied axially at the centre of the bearing to achieve a 10 mm displacement of the central area at a frequency of 15 Hz. The FE analysis of the radial test was performed for a bearing load of 10 N. With the results of these two tests, the axial and radial stiffness can be calculated as

$$k_a = F_x/\delta_x \text{ and } k_r = F_y/\delta_y. \quad (1)$$

Fig. 5 shows that the axial stiffness obtained in the FE analysis agrees well with that calculated by Wahl's correlation for turn angles above 450°. Below this value, however, theory underestimates the axial stiffness. Furthermore, the dynamic load required for a 10 mm displacement increases with lower turn angles and fewer arms.