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Real gas effects on the temperature profile of regenerators

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ABSTRACT

Real gas properties impose profound effects on regenerators. The enthalpy flow associated with the pressure dependence, abbreviated as the pressure-induced enthalpy flow, generally affects the temperature profile of regenerators and makes it far from linear at temperatures close to and below the critical point. This phenomenon has attracted great interest for a long time. In this paper, the factors in shaping the temperature profile are investigated. Different from previous studies, our analysis has revealed that the enthalpy flow associated with imperfect heat transfer and limited matrix heat capacity in the regenerator, abbreviated as the heat-associated enthalpy flow, together with the pressure-induced enthalpy flow plays the main role in shaping the temperature profile. This identification suggests a starting point for further investigation into the working mechanism at liquid-helium temperatures, and also for improvement of refrigeration efficiency in this temperature range.

1. Introduction

Real gas properties impose profound effects on regenerators Appendix A. The enthalpy flow associated with the pressure dependence, abbreviated as the pressure-induced enthalpy flow, is the significant real gas property, and generally induces serious losses and consequently decreases the refrigeration efficiency at the temperatures close to and below the critical point [1]. Regenerator temperature profiles that are far from linear have been observed since the 1990s [1–5]. This phenomenon has attracted great interest for a long time and great progresses have been achieved. Wang et al. calculated the temperature profile with a numerical method and compared that with experiment measurements [2,6]. The investigation has provided detailed information about the working mechanism of pulse tube cryocoolers and is quite comparable with experiment measurements. De Waele et al. initiated a theoretical method to solve this issue, they systematically analyzed the energy balance of the regenerator and found that heat conduction of the regenerator is important for the temperature profile [7]. Together with the investigation on the finite heat-capacity effects, heat conduction was considered to play an important role [8]. These studies [7,8] have paved the way to solve the issue of the temperature profile of regenerators working at the temperatures close to and below the critical point. Kittel studied the heat conduction in the pulse tube cryocooler systematically and pointed out that the heat conduction of the regenerator shapes the temperature profile [9]. Radebaugh et al. calculated the temperature profile with NIST numerical code REGEN [1,3], with which they have found an interesting phenomenon that the second-law COP of the regenerator at about 4 K changes little when the hot-end temperature increases in a certain range [1,10].

Similar method of energy balance [7] is taken to investigate the temperature profile in this paper, another factor is found to be important besides the heat conduction. The magnitude of heat conduction is limited at liquid-helium temperatures and is not able to keep the energy balance in practical cases. In this paper, the relationship between the pressure-induced enthalpy flow and the PV power is derived in a concise way. The magnitude of the pressure-induced enthalpy flow and the heat conduction are compared based on the investigation into cases of pulse tube cryocoolers published in open literatures. The enthalpy flow associated with imperfect heat transfer and limited matrix heat capacity in the regenerator, abbreviated as the heat-associated enthalpy flow, is estimated and its relationship with the temperature gradient is derived in detail. Note that the temperature gradient in this paper means the magnitude of temperature gradient and it is always positive or zero. Furthermore, the methods for improving the refrigerator performance are discussed.

2. General expression of the pressure-induced enthalpy flow

The enthalpy of real gases is a function of temperature as well as pressure, thus the enthalpy consists of the temperature-related

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Nomenclature

a_2	Eq. (A10)
A_w	wetted area (m^2)
COP	coefficient of performance
c_T	Eq. (2) (J/kg Pa)
d_h	hydraulic diameter (m)
$\dot{H}$	enthalpy flow (W)
h_{ht}	heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
k_T	Eq. (14) (W m/K)
m	mass (kg)
$\dot{m}$	mass flow rate (kg/s)
Pr	Prandtl number
PV	pressure–volumetric power (W)
$\dot{Q}$	cooling power or thermal loss (W)
Re	Reynolds number
u	specific internal energy (J/kg)
v	specific volume (m^3/kg)
Z	impedance magnitude (Pa s/kg)

Greek letters

β	volume expansivity (1/K)
μ	viscosity (Pa s)
ξ	conduction degradation factor
Φ	porosity of matrix

Subscripts

0	mean/ambient value
1	amplitude
c	cold end
cond	conduction
cr	cross section
dT	temperature difference
g	gas
h	hot end
m	matrix
p	pressure induced
T	temperature related
tot	total amount
w	wall

enthalpy and the pressure-induced enthalpy. The differential form of the specific pressure-induced enthalpy is expressed as:

$$dh_p = v(1 - T \cdot \beta) \cdot dp. \tag{1}$$

In this equation, the term $v(1 - T \cdot \beta)$ is a coefficient between the pressure and the enthalpy, and is notated as c_T in this paper, as given:

$$c_T = v(1 - T \cdot \beta). \tag{2}$$

In order to derive the magnitude of the pressure-induced enthalpy flow in the regenerator, a small pressure ratio is assumed, thus both the time-variation of volume expansivity and the time-variation of temperature are small and are regarded as constant during the integration in one time cycle. The specific pressure-induced enthalpy is integrated over the pressure and the mean pressure is used as the reference pressure. The time-averaged pressure-induced enthalpy flow $\langle \dot{H}_p \rangle$ is calculated with the equation

$$\langle \dot{H}_p \rangle = \frac{1}{\tau} \oint \dot{m} c_T (p - p_0) dt. \tag{3}$$

The equation is simplified as:

$$\langle \dot{H}_p \rangle = (1 - T \cdot \beta) \frac{1}{\tau} \oint \dot{V} (p - p_0) dt. \tag{4}$$

Compared with the expression of PV power [11], $\langle \dot{H}_p \rangle$ is expressed as:

$$\langle \dot{H}_p \rangle = (1 - T \cdot \beta) < pV >. \tag{5}$$

It is clear that the pressure-induced enthalpy flow is proportional to the PV power with a coefficient of $(1 - T\beta)$. In the cases of relative large pressure ratio, the volume expansivity and the temperature vary significantly with time in one time cycle. The average value of volume expansivity and temperature should be used in the calculation.

2.1. Magnitude of the pressure-induced enthalpy flow

The pressure-induced enthalpy flow usually accounts for a large percent of PV power at temperatures close to and below the critical point. As shown in Fig. 2 in Ref. [10], when the pressure is 1 MPa $(1 - T\beta)$ reaches the lowest point of −96% at 8.7 K for ^{4}He [12], and reaches as high as 78% at 4.2 K. The real gas effect of ^{3}He is smaller than that of ^{4}He. For ^{3}He at 1 MPa $(1 - T\beta)$ reaches the lowest point of −33% at 9.5 K and 69% at 4.2 K [13]. For both ^{4}He and ^{3}He $(1 - T\beta)$ invariably goes up to 1 at the temperature of zero volume expansivity. This temperature is close to the lambda-line for ^{4}He. For both ^{4}He and ^{3}He, $(1 - T\beta)$ increases with pressure in the temperature range between the point where $(1 - T\beta)$ equals 1 and the point of smallest $(1 - T\beta)$.

2.2. Variation of the pressure-induced enthalpy flow along the regenerator

Variation of the pressure-induced enthalpy flow along the regenerator is estimated in a simplified way in this part. The regenerator is assumed to be isothermal and without friction losses. The PV power varies with specific volume in isothermal parts [11], as shown in the equation

$$\frac{< pV >_c}{< pV >_h} = \frac{\upsilon_c}{\upsilon_h}. \tag{6}$$

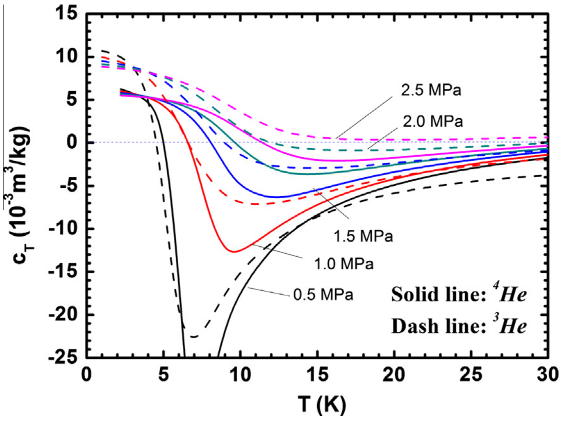

Fig. 1. Dependence of c_T on temperature for ^{4}He [12] and ^{3}He [13].