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A new approach to optimize thermoelectric cooling modules

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ABSTRACT

A theoretical investigation to optimize thermoelectric cooling modules is performed using a novel one-dimensional analytic model. In the model the optimum current, which maximizes the COP of a thermoelectric cooling module, is determined by the cooling capacity of a thermoelement, the hot and cold side temperatures, the thermal and electrical contact resistances and the properties of thermoelectric material, but not by the length of a thermoelement. The optimum length of a thermoelement can be easily obtained using the optimum current. The effects of the thermal and electrical contact resistances, the cooling capacity of a thermoelement and the cold side temperature on the maximum COP, the optimum electric current and the optimum thermoelement length are shown.

1. Introduction

Thermoelectric coolers have been increasingly employed as cooling devices in many areas such as microelectronic systems, telecommunications, superconductor systems and aerospace industry since they have the advantages of compact structure, high reliability, no vibration and direct electric energy conversion [11,9,10].

The main drawback of thermoelectric coolers is the poor coefficient of performance (COP), particularly in large capacity applications and wide temperature range applications [9,7]. Improvement of the COP of thermoelectric coolers can be achieved by means of developing new materials with a higher figure of merit, optimizing module design and fabrication, improvement of the heat exchange efficiency and employment of multi stage systems [9,7].

A thermoelectric cooling module or Peltier module is a building block for construction of thermoelectric cooling systems [4]. The COP and the cooling capacity of the module may be estimated using the conventional module theory based on one-dimensional heat balance equations, in which the thermal and electrical contact resistances are neglected [7,4]. The COP derived in the conventional module theory is determined by the hot and cold side temperatures of a module and the figure of merit of thermoelectric material [7,5], but not by the length of a thermoelement. Hence, it cannot be used to optimize the length of a thermoelement [4]. Min and Rowe [5] developed an improved model which took into account both the thermal and electrical contact resistances. Their model provides a better accuracy for modeling of thermoelectric cooling modules and proves to be very useful in analysis and optimal design of small dimension thermoelectric cooling modules [4]. Pettes et al. [6] extended the conventional module theory to account for the thermal and electrical contact resistances. They provided a procedure to calculate the optimum thermoelement length which maximizes the cooling capacity or the COP at a specified cooling capacity [6].

Usually, the design of a thermoelectric cooling system starts from a given temperature difference across the hot and cold sides of the module and the required cooling capacity [7,3]. In the conventional module theory and Min and Rowe's model the cooling capacity is a function of the hot and cold side temperatures, the length and cross-sectional area of a thermoelement, and the electric current [7,5]. An iteration procedure is necessary for the cooling capacity to meet the required value. So, it may not be convenient to use them for designing a new thermoelectric module.

In this paper, a new approach to optimize the COP of a thermoelectric cooling module for a given cooling capacity is proposed. In the approach the heat flow at any axial location on a thermoelement is expressed by the temperature at that location, the electric current, the heat flow and the temperature at the cold end of a thermoelement and the properties of thermoelectric material, but not by the distance from the cold end to that location. In Section 2 the ideal model, in which the thermal and electrical contact resistances are neglected, is presented and the optimum current and the optimum thermoelement length to maximize the COP are obtained analytically. The more realistic model, in which the contact resistances are taken into account, is described and the procedure to obtain the optimum current and the optimum thermoelement length is proposed in Section 3.

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Nomenclature

A	cross-sectional area of a thermoelement
C_i	coefficients defined by Eq. (27)
COP	coefficient of performance
I	electric current
k	thermal conductivity
L	length of thermoelement
q	heat flow for one thermoelement
r	contact resistance
T	temperature
V	electric voltage
x	coordinate defined in Fig. 1
z	figure of merit of thermoelement
Greek letters	
α	seebeck coefficient
ρ	electrical resistivity
τ	dummy variable
Subscripts	
c	cold side
$crit$	critical
e	electrical
h	hot side
$ideal$	neglecting thermal and electrical contact resistances
j	interface between thermoelement and metal strip
m	mean
max	maximum
min	minimum
opt	optimum
t	thermal

2. Ideal model

Fig. 1 shows a schematic diagram of a thermocouple, which is the basic unit of a thermoelectric cooling module [4]. It consists of a p-type thermoelement, an n-type thermoelement and metal strips. The thermoelements are connected electrically in series and thermally in parallel by highly conducting metal strips [9,4]. For a p-type thermoelement the heat due to the Peltier effect flows in the direction of electric current, but the heat flows in the opposite direction of electric current for an n-type thermoelement. The heat flow from the cold side to the hot side is defined to be positive. T_c and T_h are the cold side temperature and the hot side temperature of the thermocouple, respectively.

Heat flow along a thermoelement, q, and electric current, I, can be written as follows [8,1]:

$$q = -kA\frac{dT}{dx} \pm \alpha IT \tag{1}$$

$$I = -\frac{A}{\rho}\left(\frac{dV}{dx} \pm \alpha\frac{dT}{dx}\right) \tag{2}$$

where α, ρ and k are the Seebeck coefficient, electrical resistivity and thermal conductivity of a thermoelement, respectively. A denotes the cross-sectional area of a thermoelement and V denotes the electric voltage. The sign in front of α is (+) for a p-type thermoelement and (−) for an n-type thermoelement.

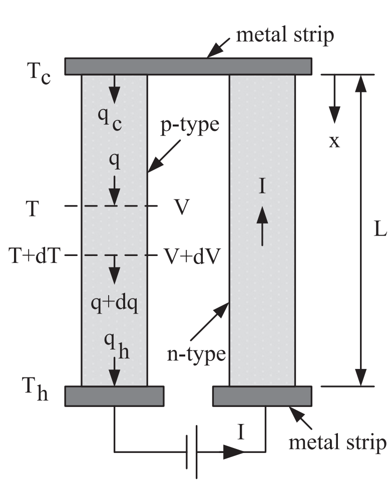

Fig. 1. Schematic diagram of an ideal single thermocouple Peltier module.

We assume that the electrical and thermal contact resistances between the thermoelements and the metal strips are negligible. If we further assume that α, ρ and k for both p-type and n-type materials are the same, the heat flow and temperature distribution for the p-type thermoelement will be the same as those for the n-type thermoelement. Only the heat flow along the p-type thermoelement will be considered here. α is assumed to be independent of temperature throughout the paper.

For an infinitesimal length of the p-type thermoelement shown in Fig. 1, the one-dimensional energy balance is given as follows:

$$dq + IdV = 0 \tag{3}$$

By combining Eqs. (2) and (3) we can obtain the following expression.

$$d(q - \alpha IT) = \frac{\rho I^2}{A}dx \tag{4}$$

Substituting the expression for dx which can be obtained from Eq. (1) into Eq. (4) gives the following relation.

$$(q - \alpha IT)d(q - \alpha IT) = -\rho k I^2 dT \tag{5}$$

Integrating Eq. (5) over the thermoelement gives following expression.

$$\frac{1}{2}[(q_h - \alpha IT_h)^2 - (q_c - \alpha IT_c)^2] = -I^2\int_{T_c}^{T_h}\rho k dT \tag{6}$$

If ρ and k are assumed to be constant, Eq. (6) can be arranged for the heat flowing through the hot end of a thermoelement.

$$q_h = \alpha IT_h - \sqrt{(q_c - \alpha IT_c)^2 - 2\rho k I^2(T_h - T_c)} \tag{7}$$

The sign in front of the root in Eq. (7) can be (+) or (−). The reason why (−) is chosen here is shown in Appendix A. For the value inside the root to be non-negative, the electric current should be equal to or larger than the minimum current defined as below.

$$I_{\min} = \frac{q_c}{\alpha T_c - \sqrt{2\rho k(T_h - T_c)}} \tag{8}$$

In other words, $I_{\min}$ is the minimum electric current required for the cooling capacity of a thermoelement to be q_c for given T_c, T_h and thermoelectric material properties.