Cryogenics 51 (2011) 168–172

Contents lists available at ScienceDirect

Cryogenics

journal homepage: www.elsevier.com/locate/cryogenics

Impedance match for Stirling type cryocoolers

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ARTICLE INFO

Article history:
Received 22 October 2010
Received in revised form 30 December 2010
Accepted 5 January 2011
Available online 9 January 2011

Keywords:
Linear compressor
Acoustic impedance
Impedance match
Efficiency

ABSTRACT

Impedance match in Stirling type cryocoolers is important for the compressor efficiency and available acoustic power. This paper generalizes the basic principles concerning the efficiency and acoustic power output of the linear compressor. Starting from basic governing equations and mainly from the viewpoint of energy balance, the physical mechanisms behind the principles are clearly shown. Specially, this paper focuses on the impedance match for an existing compressor, where the current limit and displacement limit should also be taken into consideration when selecting a suitable impedance. Some case studies based on a commercial compressor are also provided for a deep understanding.

1. Introduction

Optimizing the efficiencies of Stirling cryocoolers and Stirling type pulse tube cryocoolers is critical for both space applications and many civil applications. With high frequency oscillating flow inside, these cryocooler systems can be grouped into thermoacoustic systems [1]. From the acoustic viewpoint, the impedance match between the cold-head and compressor is very important for optimizing the efficiency of the compressor as well as maximizing the power capability of the compressor. A few papers have discussed the impedance match for optimizing the efficiency of the compressor [2,3]. However, a systematic discussion about the general rules is lacking, which is the purpose of this paper. The following sections start from the basic governing equations of the linear compressor and perform an in-depth theoretical analysis. Case studies are also provided for a better understanding.

As a matter of fact, there are two different ways when designing the cooler system. One is to design a compressor to match an existing cold-head, which is relatively straightforward but the guiding rule about optimum efficiency should be kept in mind. The other is to find a cold-head to match an existing compressor, which is more important for those who themselves do not have the facility to manufacture a linear compressor and the cost is too high to order a customer-built model. The case studies will mainly focus on the later issue and take the pulse tube cryocooler as an example. A typical illustration of the Stirling type pulse tube cryocooler is shown in Fig. 1. Note that the same analysis also naturally applies to Stirling cryocoolers.

2. The governing equations and some basics

The governing equations in complex frequency domain for a linear compressor are

$$\hat{E} - BL\hat{u} = \hat{I}(R_e + i\omega L_e) \quad (1)$$

$$\hat{I}BL = \hat{p}A + k\frac{\hat{u}}{i\omega} + R_m\hat{u} + mi\omega\hat{u}, \quad \frac{\hat{u}}{i\omega} = \hat{x} \quad (2)$$

Here ^ denotes the complex value containing information on both amplitude and phase of oscillating parameters, which is a conventional method used in the study of alternating electric circuit and acoustics. In Eq. (1), $\hat{E}$ is applied electric voltage, BL is force factor, $\hat{u}$ is the piston velocity, $\hat{I}$ is the current, R_e and L_e are resistance and inductance of the motor coil, ω is angular frequency, i denotes the imaginary part. In Eq. (2), $\hat{p}$ is the pressure wave at the piston front side which drives the cold-head to cool down, A is the piston area, k is spring constant which includes mechanical, magnetic (in case of moving-magnet type motor) and backside gas spring effects, R_m is sometimes called the damping coefficient or the mechanical resistance, m is the moving mass of the compressor and $\hat{x}$ is piston displacement. These equations may seem too simple if we consider many non-linear effects inside the compressor [4], such as the eddy current loss, non-constant BL and clearance losses. Even the equivalent electric circuit may be different in a moving-magnet type motor [5]. However, our purpose here is to build the framework of principles to design a well-matched system and from our

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doi:10.1016/j.cryogenics.2011.01.002

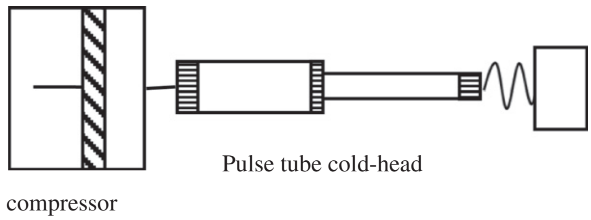

Fig. 1. Illustration of a Stirling type pulse tube cryocooler.

experience, the following analyses based on these equations suffice to provide a useful insight into the mechanism of impedance match and thus lead to a rather good initial design of the practical systems.

Multiplying both sides of Eq. (1) with conjugate of current $\hat{I}^*$, and using 1/2 Re operator (Re means taking real part) to calculate cycle-averaged power flow

$$\begin{aligned}\frac{1}{2}\mathrm{Re}(\hat{E}\hat{I}^*) &= \frac{1}{2}\mathrm{Re}(\hat{I}^* BL\hat{u}) + \frac{1}{2}\mathrm{Re}(\hat{I}(R + i\omega L)\hat{I}^*) \\ &= \frac{1}{2}\mathrm{Re}(BL\hat{u}\hat{I}^*) + \frac{1}{2}|\hat{I}|^2 R_e \end{aligned} \tag{3}$$

The physical meaning of this equation is very clear, which is that the electrical power input is partly used for electro-magnetic force to do work and partly dissipated into heat via the resistance. Similarly, multiplying both sides of Eq. (2) with conjugate of piston velocity $\hat{u}^*$ and use 1/2 Re operator, we have

$$\begin{aligned}\frac{1}{2}\mathrm{Re}(\hat{I}BL\hat{u}^*) = {} & \frac{1}{2}\mathrm{Re}(\hat{p}A\hat{u}^*) + \frac{1}{2}\mathrm{Re}\left(k\frac{\hat{u}}{i\omega}\hat{u}^*\right) + \frac{1}{2}\mathrm{Re}(R_m\hat{u}\hat{u}^*) \\ & + \frac{1}{2}\mathrm{Re}(mi\omega\hat{u}\hat{u}^*)\end{aligned} \tag{4}$$

The second and the fourth term on the right hand side are both equal to zero. Using the thermoacoustic terminology of acoustic impedance

$$Z = \hat{p}/\hat{U} \tag{5}$$

where $\hat{U}$ is volumetric flow rate defined as the piston velocity multiplied by the piston area, Eq. (4) becomes

$$\begin{aligned}\frac{1}{2}\mathrm{Re}(\hat{I}BL\hat{u}^*) &= \frac{1}{2}\mathrm{Re}(\hat{p}A\hat{u}^*) + \frac{1}{2}\mathrm{Re}(R_m\hat{u}\hat{u}^*) \\ &= \frac{1}{2}|\hat{u}|^2(\mathrm{Re}(\mathrm{ZAA}) + R_m)\end{aligned} \tag{6}$$

Here we refer to Re(ZAA) as the acoustic resistance to distinguish it from the mechanical resistance R_m. The physical meaning of this equation is also very clear: the work done by electro-magnetic force is consumed by the acoustic resistance Re(ZAA) in the form of acoustic power and the mechanical resistance R_m. There is a delicate difference between the cold-head impedance Z_{ptc} and the impedance Z seen by the compressor piston. The difference could be strongly influenced by the void volume between the cold-head and piston and is important for the impedance match. As seen through this equation, the definition of R_m is also very delicate. Besides the useful work for the cold-head, this equation actually means that many other losses, such as the leakage through the clearance, the friction between piston and cylinder if the clearance is not well kept and the hysteresis losses backside of the compressor, have been generalized into this coefficient.

So from Eqs. (3) and (6), the electrical power input is divided into three parts: one is dissipated in the electric resistance, one is dissipated in the mechanical resistance, and the remaining part goes to the acoustic load for useful work. Properly handling the balance between them is the key to both compressor efficiency and the available acoustic power.

Combining both Eqs. (2) and (3) leads to

$$\begin{aligned}\frac{1}{2}\mathrm{Re}(\hat{E}\hat{I}^*) &= \frac{1}{2}\mathrm{Re}\left(\frac{\hat{I}(BL)^2}{\mathrm{Z\hat{A}A} + k/i\omega + R_m + i\omega m}\hat{I}^*\right) + \frac{1}{2}|\hat{I}|^2 R_e \\ &= \frac{1}{2}|\hat{I}|^2 \frac{(BL)^2(\mathrm{Re}(\mathrm{ZAA}) + R_m)}{(\mathrm{Re}(\mathrm{ZAA}) + R_m)^2 + (\mathrm{Im}(\mathrm{ZAA}) - k/\omega + \omega m)^2} + \frac{1}{2}|\hat{I}|^2 R_e\end{aligned} \tag{7}$$

I_m means taking the imaginary part. For the cold-head, the imaginary part of the load impedance is negative. As already deduced in Ref. [2], one of the basic requirements for high efficiency of the system is that

$$\mathrm{Im}(\mathrm{ZAA}) - k/\omega + m\omega = 0 \tag{8}$$

which means that the mass is mechanically resonated by spring forces with a spring constant equal to $k - \omega\mathrm{Im}(\mathrm{ZAA})$. $-\omega\mathrm{Im}(\mathrm{ZAA})$ is the gas spring at the front side of the piston, which contributes a very large proportion of the whole spring force in a normal design.

Rewriting Eq. (2) into the form

$$\hat{I}BL = (\mathrm{Re}(\mathrm{ZAA}) + R_m)\hat{u} + i(\mathrm{Im}(\mathrm{ZAA}) - k/\omega + m\omega)\hat{u} \tag{9}$$

And combining it with Eq. (8), we can see that the motor current is in-phase with the velocity of the piston. This in-phase relationship is well known and reflects that in the resonant state the current amplitude is minimized in producing the same amount of work and Joule loss in the motor coil is thus reduced. Off-resonance only means that part of the electro-magnetic force acts as a kind of spring force (either positive or negative) without doing any work. At this mechanically-resonant point, Eq. (7) further reduces to

$$\frac{1}{2}\mathrm{Re}(\hat{E}\hat{I}^*) = \frac{1}{2}|\hat{I}|^2\frac{(BL)^2}{\mathrm{Re}(\mathrm{ZAA}) + R_m} + \frac{1}{2}|\hat{I}|^2 R_e \tag{10}$$

The first item on the right hand side is actually another form of the electro-magnetic work. Starting with these equations, we will discuss how to maximize the compressor efficiency and acoustic power output. Unless otherwise stated, the following discussion is based on the assumption that mechanical resonance is achieved by satisfying Eq. (8).

3. Maximizing the efficiency and acoustic power output of a compressor

3.1. Impedance match for maximizing the efficiency

The compressor efficiency is defined as output acoustic power W_a, i.e., the power dissipated by the acoustic resistance, divided by input electric power W_e.

$$\eta = \frac{W_a}{W_e} = \frac{\frac{1}{2}|\hat{u}|^2\mathrm{Re}(\mathrm{ZAA})}{\frac{1}{2}\mathrm{Re}(\hat{E}\hat{I}^*)} \tag{11}$$

From Eqs. (6) and (10), it is very clear that the acoustic resistance Re(ZAA) must be kept within a certain range for a high compressor efficiency. If Re(ZAA) is too small, although most of the electric power will be converted into electro-magnetic work (i.e. the first item on the right hand side of Eq. (10)), most of the work will be consumed by the mechanical resistance, not by the acoustic load, as expressed through Eq. (6). However, if Re(ZAA) is too large, then from Eq. (10), most of the input electric power will be dissipated by the coil resistance. So an appropriate intermediate value must be selected for Re(ZAA) for the sake of a high efficiency operation of the compressor. Although Refs. [2,3] has determined the correct value of Re(ZAA) for a maximum-efficiency operation of