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Constructal design of stack filled with parallel plates in standing-wave thermo-acoustic cooler

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ABSTRACT

The optimal design for a stack filled with parallel plates in a standing-wave thermo-acoustic cooler was studied for fixed cross-sectional area constraints by using the constructal principle in this paper. The relationship between the cooling load and the plate spacing is derived. These expressions for the optimal plate spacing or the channel size and the optimal plate number are obtained. The results obtained herein shows that in the stack design of a thermo-acoustic cooler, the plate spacing and plate-number should been suitably selected based on the value of L_0 and ω so as to obtain the maximum cooling load.

1. Introduction

The period compressions and expansions of the gas in an acoustic wave combined with heat exchange with external reservoirs generate a rich variety of thermo-acoustic processes [1,2]. A new class of prime-movers and coolers called as thermo-acoustic devices make use of these effects [3–5]. With the development of the electronic apparatuses and the superconductivity application technique, a variety of coolers with good performances are needed. The traditional coolers cannot completely meet these needs in new features. Many engineers have been developing new types of coolers. The thermo-acoustic cooler without moving parts has the advantages of high reliability and non-pollution. It has captivated many engineers.

The principle parts of a thermo-acoustic cooler are the stack and two adjacent heat exchangers. The acoustic wave carries the working gas back and forth within these components. A longitudinal pressure oscillating in the sound channel induces a temperature oscillation in time with angular frequency ω. The energy conversion between the heat and the sound occurs in the stack, which is an absolutely necessary part of the thermo-acoustic cooler, influencing the efficiency of the device. Therefore the design of the stack is a key factor in order to develop the thermo-acoustic cooler with high efficiency.

There is an old history of trying to explain the forms of nature – why leaf has nerves, why a flower has petals and so on. Geometry has focused on explaining form, and has contributed too much of the knowledge inherited from antiquity. Constructal theory established by Professor Adrian Bejan is a principle-based method of constructing machines, which attain optimally their objective [6–10]. It provides a conceptual framework for predicting form and evolution of form and for modeling natural or engineered systems. Flow systems acquire configuration and achieve high performance.

The constructal principle is applied as follows. The elemental structure is defined, determined and optimized. The first problematic question is to defining and determining the ideal proportions adapted to the available elemental system, which has been defined and determined. The next step consists in uniting several elemental structures in a network.

The constructal-theory is a powerful tool for investigation of practical engineering design [11]. Bejan and Sciubba considered this problem for an array of parallel plates with application to the cooling of electronic systems [12]. Using the intersection of asymptotes method, they obtained expressions for the optimal plate spacing to channel length ratio and the maximum heat transfer per unit volume in term of a dimensionless parameter which is now referred to as the Bajan number. Muzychka applied the approximate analysis method of Bejan and Sciubba to heat exchangers with several other channel-shapes [13]. Many authors have studied the heat transfer models of porous media [14], heat exchanger [15], cooling of electronics [16] by constructal theory.

There are many optimization strategies in the optimization design of a cooling device, such as the cooling load, the coefficient of performance, the exergetic efficiency, and so on. The cooling load, which decides the refrigerating capacity of a cooler, is an important performance parameter for a cooling cycle. The cooling effect is

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good when the cooling load is large. Therefore, the need to develop the performance of the cooling devices is one reason why the maximum cooling load is discussed.

The stack in a thermo-acoustic cooler (or engine) consists of porous material (such as parallel plates or circular tubes). So far the optimal design of the stack has rarely been studied. Their structure is determined by repeated experiment or practical experience. In the present work, we discuss the issues that corners around the design of an actual stack filled with parallel plates in a standing-wave thermo-acoustic cooler. Bathed on the actual situation many simplistic assumptions are commonly adopted. The optimal structure of the stack in a standing-wave thermo-acoustic cooler is analyzed by using the constructal optimization. The expressions for the optimal plate spacing and the optimal plate number for an array of parallel plates are obtained in this paper. Numerical example leads to the relation between the optimal plate spacing and other parameters. The optimal dimension represents a basic constructional unit for built up the stack. These optimal scales are then applied to arrays of passages to determine the maximum cooling load of the cooler system. The results obtained herein will be useful for the optimal design of real stack in a standing-wave thermo-acoustic cooler.

2. The cooling load of the system

Fig. 1 depicts a schematic diagram of the main structure of a standing-wave thermo-acoustic cooler. The thermo-acoustic cooling part which consists of two heat exchangers and a stack is mounted inside a circular tube called standing-wave resonator. The tube inner radius is R and length is $\frac{\lambda}{4}$ or $\frac{\lambda}{2}$, where λ is wavelength. The stack made of an array of parallel plates. Fig. 2 shows the stack in a cross-section. The cross-sectional area πR^2 is fixed. We define the pacing between two plates as $2y_0$ and the width, thickness of jth solid plate as L_j, $2L_0$, respectively. These plates may be arranged in such a manner that y_0 and L_0 are identical on all plates. It may be seen that plate-number of the stack is

$$n = \frac{2R}{2L_0 + 2y_0} = \frac{R}{L_0 + y_0} \tag{1}$$

First we consider the single sound-channel which consists of jth plate and $(j+1)$th plate. The cooling load for jth sound-channel may be written as [17,18]

$$\dot{Q}_{2j} = \dot{Q}_{cond} + \dot{Q}_{dyn} + \dot{Q}_{pr} + \dot{Q}_{st} \tag{2}$$

where $\dot{Q}_{pr}$ and $\dot{Q}_{st}$ are the progressive and standing components of the cooling load, $\dot{Q}_{dyn}$ expresses the heat flux caused by convective transport of the working gases, and is the heat loss in the cooler,

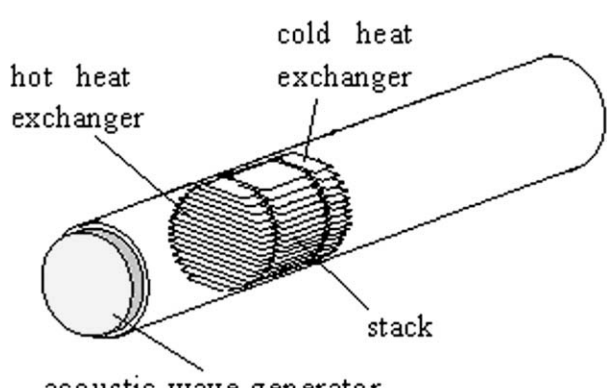

Fig. 1. A schematic diagram of the main structure of a standing-wave thermo-acoustic cooler.

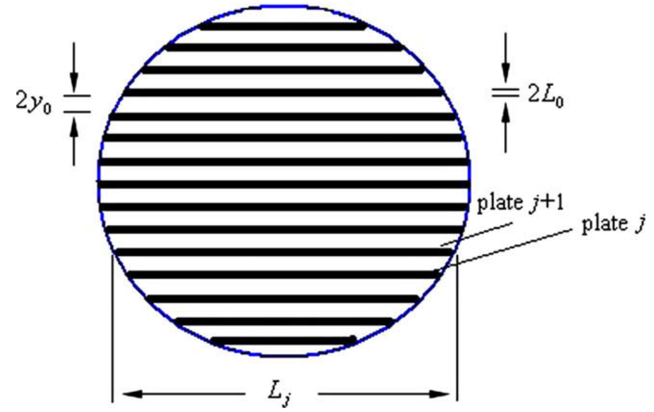

Fig. 2. A schematic diagram of the stack with finite cross-sectional area.

$\dot{Q}_{cond}$ is the heat loss due to the heat transfer from the hot end to cold end through the working gases and the solid plates.

We assumed the heat capacity of the fluid is small enough compared with that of the solid, or the ratio of heat capacity of the fluid to that solid plate vanishes ($\varepsilon = \frac{c_{pf}}{c_{ps}} \approx 0$). For a standing-wave thermo-acoustic cooler, $\dot{Q}_{pr}$ and $\dot{Q}_{cond}$ can be neglected [18]. Eq. (2) may be rewritten as

$$\dot{Q}_{2j} \approx \dot{Q}_{dyn} + \dot{Q}_{st} \tag{3}$$

with

$$\dot{Q}_{dyn} = \rho_m c_p \frac{1 - R_e(f_v)}{|1 - f_v|^2} I_m(g_D) \cdot \frac{1}{2} \frac{|u_1|^2}{\omega} \frac{dT_m}{dx} \cdot (2L_j y_0)$$

$$\dot{Q}_{st} = -\beta T_m I_m(F_s g) \frac{1}{2} |p_1||u_1| \sin\theta \cdot (2L_j y_0)$$

$$g_D = \frac{(f_k - f_v^+) - \mathrm{i}(1+\sigma) I_m(f_v)}{(1 - \sigma^2)[1 - R_e(f_v)]}$$

$$g = \frac{f_k - f_v^+}{(1+\sigma)(1 - f_v^+)}$$

$$f_k = \frac{\tanh[(1+\mathrm{i}) y_0/\delta_k]}{(1+\mathrm{i}) y_0/\delta_k}$$

$$f_v = \frac{\tanh[(1+\mathrm{i}) y_0/\delta_v]}{(1+\mathrm{i}) y_0/\delta_v}$$

$$F_s = \frac{1}{1 + \varepsilon f_k}$$

where σ is the fluid Prandtl number, $\delta_k = \sqrt{\frac{2K}{\rho_m c_p \omega}}$ and $\delta_v = \sqrt{\frac{2\mu}{\rho_m \omega}} = \sqrt{\sigma}\delta_k$ are the thermal penetration depth and the viscous penetration depth in the fluid, K, μ and ω are the thermal conductivity, dynamic viscosity of the fluid and the angular frequency, f_k and f_v are the thermal dissipation function and the viscous dissipation function, ρ_m, T_m, β and c_p are the mean density, the mean temperature, thermal expansion coefficient and the isobaric heat capacity per unit mass of the fluid, $|p_1|$ and $|u_1|$ are the amplitude of the pressure oscillation and the velocity oscillation, θ is the phase difference between the pressure oscillation and the velocity oscillation, $R_e()$ and $I_m()$ indicate the real part and imaginary part of complex number, the superscript "+" indicates a complex conjugate of a quantity, x is the position along sound propagation.

In this paper, we make the "boundary-layer" approximation. We will limit y_0 to $1 \leqslant \frac{y_0}{\delta_k} \leqslant 2$. In this case, we have [1]

$$f_v \approx \frac{(1-\mathrm{i})\delta_v}{2y_0} \tag{4}$$

$$f_k \approx \frac{(1-\mathrm{i})\delta_k}{2y_0} \tag{5}$$