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A simple method to determine the frequency of engine-included thermoacoustic systems

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Abstract

Frequency determination is important for the simulation of a thermoacoustic system consisting of a thermoacoustic engine. Based on the characteristics of linear acoustics, this article proposes a simple method for frequency calculation through numeric investigation. According to the method, frequency value can be decided if it leads to an inflexion point of the amplitude of volume flow rate, which is also a local minimum, most close to the volume flow rate node boundary. Compared with experimental data, the method proves to be very reliable. Besides, a concept of virtual tube is also proposed for frequency determination of thermoacoustic systems with no apparent geometrically-closed end.

Keywords: Thermoacoustic engine; Frequency; Acoustic node

1. Introduction

The thermoacoustic engines can generate acoustic power from heat input without the need of moving components, which may find important applications in areas such as natural gas liquefaction when they are coupled with pulse tube coolers [1] or electric power generation when they are coupled with motors [2], etc. Simulations of the systems are very helpful in searching for better operating conditions or better configurations. One special thing related to the simulation of the thermoacoustic engine is to calculate the oscillation frequency which is spontaneously decided by the acoustic boundary conditions inside the system. The frequency value greatly influences the important coefficients in the governing dynamic and energy conservation equations for all components, while solving these equations leads to the results on the thermal performance of the system. Thus, it is of top priority that the approximate frequency value should be determined first and refined together with the processes of the other calculations. So far, there are limited literatures reporting on this issue and there is no clear or satisfying explanation of how the frequency is calculated. The famous program DELTAE [3] gives the system simulation algorithms based on the thermoacoustic control equations. Although frequency guess is mentioned but no clear clue can be found about how the frequency value is effectively determined. Tu et al. have proposed using network method to calculate the frequency [4]. However, the method they use could lead to a complex frequency value, which is interesting but has no solid physical background. Meanwhile, how to calculate the system thermal performance with a complex frequency value remains unclear.

This article will introduce a simple and experimentally-verified method to calculate the frequency, which is based on the characteristics of linear acoustics and can be easily implemented in the numeric program. Next section gives the physical background of the method, then, numeric calculations are performed to setup a more realistic criterion. Besides, a "virtual tube" concept is introduced for determining frequency of the system with no practical geometrical closed end. Then the third section gives some comparisons between calculation and experimental results. Finally, conclusion is made.

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2. Frequency determination methodology

2.1. Physical background of possible frequency determination criteria

To show the physics behind our computation algorithm, we first take a look at the resonance tube inside a standing-wave engine with right end closed as shown in Fig. 1. Based on the linear thermoacoustic theory [5], the underlying wave propagation equations in complex form are

$$\frac{\mathrm{d}\widehat{U}_1}{\mathrm{d}x} = -\frac{\mathrm{i}\omega A[1+(\gamma-1)f_k]}{\gamma P_0}\hat{p}_1 \tag{1}$$

$$\frac{\mathrm{d}\hat{p}_1}{\mathrm{d}x} = -\frac{\mathrm{i}\omega\rho_0}{A(1-f_\mu)}\widehat{U}_1 \tag{2}$$

where $\hat{p}, \widehat{U}, \omega, A, \gamma, P_0, \rho_0$ are complex pressure wave amplitude, complex volume flow rate amplitude, angular frequency, cross-sectional area, specific heat ratio, mean pressure and mean density, respectively. f_μ, f_k are functions related to the frequency, flow channel shape, dimension and physical properties of the working gas, etc. Subscript 1 means first-order magnitude.

At the closed right end which is a volume flow rate node, $|\widehat{U}_1|$ is exactly zero. According to Eq. (2), the gradient of $|\hat{p}_1|$ is also zero, which implies that pressure wave amplitude is locally maximum by considering the characteristics of sinusoidal function. This, according to Eq. (1), also indicates that absolute value of the gradient of $|\widehat{U}_1|$ is a local maximum. These analyses imply several possible criteria that may be used to judge whether a frequency value is suitable. They are: (a) $\widehat{U}_1$ equals zero; (b) gradient of $|\hat{p}_1|$ is zero; (c) absolute value of the gradient of $|\widehat{U}_1|$ is locally maximum at the volume flow rate node. In fact, these criteria are too ideal to be used. As addressed before, the frequency guess should be done first in the numerical calculation when the energy equilibrium is not reached. It is impossible to exactly use these criteria at the beginning of system simulation. However, on the other hand, they do provide some clues as to how to setup the criteria for frequency determination. Below we will resort to numeric investigations to find possible criteria.

2.2. Numeric investigation of possible frequency determination criteria

With our experimental system of a standing-wave engine as illustrated in Fig. 1, whose geometrical configurations are briefly listed in Table 1, we have done some numeric investigation for frequency determination. Linear thermoacoustic model is used which gives the following control equations for all components [6]:

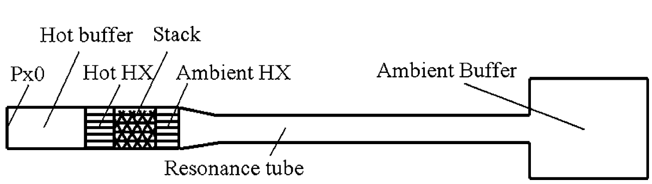

Fig. 1. Illustration of a standing-wave thermoacoustic engine in our lab, the brief geometry can be found in Table 1.

Table 1
Details of the main components of the standing-wave thermoacoustic engine

Components	Dimension
Hot reservoir	i.d. 80 mm, length 0.15 m
Hot heat exchanger (HX)	50 mm long with about 0.21 m^2 heat exchange area with gas
Stack	i.d. 79.8 mm, length 0.114 m
Ambient heat exchanger	39 mm long with about 0.29 m^2 heat exchange area with gas
Resonance tube	i.d. 50 mm, length 1.9 m
Ambient buffer	i.d. 150 mm, length 0.35 m

$$\frac{\mathrm{d}\widehat{U}}{\mathrm{d}x} + R_1\hat{p} - R_3\widehat{U} = 0 \tag{3}$$

$$\frac{\mathrm{d}\hat{p}}{\mathrm{d}x} + R_2\widehat{U} = 0 \tag{4}$$

$$\frac{\mathrm{d}H}{\mathrm{d}x} = Q, \quad H = c_1 + c_2\frac{\mathrm{d}T_x}{\mathrm{d}x} \tag{5}$$

where T_x is mean temperature of the gas, H is the total energy including enthalpy flow integrated over the cross-sectional area and static heat conduction, Q is the heat input to the gas from the outside of the system (e.g. heat input at the heater block). R_1, R_2, R_3, c_1, c_2 are complex coefficients, whose details can be found in Ref. [5].

After initializing the system with an assumed temperature gradient across the stack, we can start the calculation with a guessed pressure wave amplitude P_{x0} (0.15 MPa, which is lower than final value) and zero value of volume flow rate at the left end of the hot buffer. With pre-defined x-axis and the system discretized into N axial one-dimensional segments, the distributions of volume flow rate and pressure wave can be calculated in the form of transfer matrix from the left to the right:

$$\begin{bmatrix}\hat{p}_n\\ \widehat{U}_n\end{bmatrix} = \begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}\begin{bmatrix}\hat{p}_{n-1}\\ \widehat{U}_{n-1}\end{bmatrix} \tag{6}$$

where frequency-dependent B comes from the solution of Eqs. (3) and (4). It should be noted that no thermal calculations based on Eq. (5) is required at this moment. Fig. 2 gives the calculated amplitude distribution of volume flow rate along the system axis with different frequencies. The low density of points is intentionally selected for easiness of illustration. It is found that when the frequency value passes through the value of 45 Hz (close to the experimental value 45.8 Hz), there appears an inflexion point near the closed end for the value of $|\widehat{U}_1|$ which is locally minimum although the absolute value of the minimum $|\widehat{U}_1|$ is not zero because energy equilibrium (e.g. acoustic power) has not been reached yet. If the frequency value is 40 Hz, no inflexion point appears and if the frequency value is 50 Hz, the inflexion point is farther away from the right