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A thermodynamic model based on exergy flow for analysis and optimization of pulse tube refrigerators

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Abstract

A thermodynamic model based on exergy flow through pulse tube refrigerators (PTRs) is developed. An exergetic efficiency parameter representing the losses in the pulse tube itself is proposed. The effects of control parameters representing a general phase shifter and their effect on the system performance are discussed. Analytical solutions representing important parameters in the design of PTRs such as the load curve, cooling power and efficiency in terms of basic system input parameters are developed. It is shown that the analytical model is powerful and convenient for optimization of PTRs and in quantifying its operational bound and important losses. Results indicating a compromise between cooling power and efficiency in PTRs under certain conditions are presented and discussed.

Keywords: Pulse tube; Cryocoolers; Thermodynamics; Cycle optimization; Exergy analysis

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1. Introduction

Pulse tube refrigerators (PTRs) play an important role in satisfying the need for cryogenic cooling of space-based infrared detectors as well as electronics requiring coolers with high reliability, low vibration, and high efficiency. The conventional orifice pulse tube refrigerators (OPTRs) rely on a simple phase-shifting mechanism at the orifice where the mass flow rate and pressure are in phase. The thermodynamics of OPTRs has been under study by several investigators [1–5], to just name a few. Exergy analysis is a powerful method for the design of PTRs and for quantifying the losses in the refrigerators [6–8]. Exergy flow and analysis in PTRs for each component shows how the input exergy provided by the power input to the compressor is destroyed as the working fluid goes through its cyclic motion in the system. Recently implementation of more effective phase-shifting mechanisms has resulted in the developments of high efficiency PTRs approaching the efficiency of Stirling refrigerators [9]. In this paper we concentrate on PTRs and use the exergy method for thermodynamic analysis and optimization of the refrigerators assuming that a controlled phase-shifting mechanism exists. In this manner, we find thermodynamic bounds for cooling capacity and efficiency of the refrigerator. In addition, cooling capacity and efficiency are obtained in terms of important dimensionless numbers convenient for quantifying the important losses and performance evaluation of PTRs at the system level.

Exergy, like energy and entropy, is a property of the state of a system and measures the departure of the state of the system from the state of the environment. In application to PTRs, for each component, considering one channel heat transfer between the system and a thermal reservoir at the temperature T_R and one channel of inlet and exit mass transfer, the exergy balance can be written as [6,7]

$$\langle \dot{E}_D \rangle = \langle \dot{M}e \rangle_i - \langle \dot{M}e \rangle_e - \langle \dot{W} \rangle + \left\langle \left(1 - \frac{T_o}{T_R}\right) \dot{Q} \right\rangle \tag{1}$$

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Nomenclature

$\langle\,\rangle$	indicates integration over a cycle
A	area of regenerator
B	dimensionless variable, see Eq. (27)
$\overline{C}$	flow conductance
C_p	heat capacity at constant pressure
$\dot{E}_{\mathrm{D}}$	rate of exergy destruction
e	specific exergy
f	frequency
h	specific enthalpy
$\dot{H}$	enthalpy flow rate
K	effective thermal conductivity of regenerator
L	length of regenerator
$\dot{m}$	amplitude of mass flow rate
$\dot{M}$	mass flow rate
Mr	mass flow rate amplitude ratio across regenerator
P	pressure
p	pressure amplitude
Pr	pressure amplitude ratio across regenerator
$\dot{Q}$	rate of heat transfer
s	specific entropy
sl	slope of load curve
T	temperature
V	volume of pulse tube
$\overline{V}$	average effective void volume of regenerator
$\dot{W}$	input power
X	cosine of phase angles
Y	sine of phase angles

Greeks

θ	phase angle for pressure
η	efficiency
ψ_1	dimensionless number, $\dot{m}_2/\overline{C}p_1$
ψ_2	dimensionless number, $\overline{V}\omega/\overline{C}$
ϕ	phase angle for mass flow rate
λ	regenerator ineffectiveness
ε	pulse tube irreversibility parameter
ω	angular frequency
γ	specific heat ratio

Subscripts

a	average
c	cold
co	no load
cond	conduction
cn	cold side normalized
e	exit
ex	exergetic
i	inlet
j	general inlet or exit of components
n	normalized
o	environment
opt1	optimum for cooling capacity
opt2	optimum for efficiency
p	pressure
PT	pulse tube component
R	reservoir
th	thermal
1	hot side of regenerator, see Fig. 1
2	cold side of regenerator, second law, see Fig. 1
3	hot side of pulse tube, see Fig. 1
0, 1, 2, 3, 4	used with sine and cosine of phase angles, see Appendix A

where e is the specific exergy carried with mass, $\dot{E}_{\mathrm{D}}$ is the rate of exergy destruction in the component, and $\dot{Q}$ is the rate of heat transfer. For a single-component working fluid, such as helium, in the absence of chemical, kinetic, or potential exergy, the specific exergy can simply be written as

$$e = h - h_{\mathrm{o}} - T_{\mathrm{o}}(s - s_{\mathrm{o}}) \tag{2}$$

where h is the specific enthalpy and s is the specific entropy and subscript "o" denotes the environment. Substituting Eq. (2) into Eq. (1) and using the vanishing average mass flow rate over a cycle yields

$$\langle \dot{E}_{\mathrm{D}} \rangle = \langle \dot{M}h \rangle_{\mathrm{i}} - \langle \dot{M}h \rangle_{\mathrm{e}} - T_{\mathrm{o}}(\langle \dot{M}s \rangle_{\mathrm{i}} - \langle \dot{M}s \rangle_{\mathrm{e}}) - \langle \dot{W} \rangle + \left\langle \left(1 - \frac{T_{\mathrm{o}}}{T_{\mathrm{R}}}\right)\dot{Q} \right\rangle \tag{3}$$

Assuming the ideal gas law is valid and thermophysical properties are constant, the enthalpy and entropy in the above equation can be written, respectively as

$$h = C_p(T - T_{\mathrm{o}}) \tag{4}$$

$$s = C_p \ln(T/T_{\mathrm{o}}) - R \ln(P/P_{\mathrm{o}}) \tag{5}$$

2. Exergy based thermodynamic analysis

Fig. 1 shows important components of a PTR used for analysis in this study. Exergy comes into the system at the driver side and is destroyed as it moves into the system. The product exergy is delivered to the cold reservoir at T_{c}. We assume for purposes of analysis that the pressure and mass flow rate at any location at the inlet and exit of each component are given by

$$\dot{M}_j = \dot{m}_j \mathrm{Cos}(\omega t + \phi_j) \tag{6}$$

$$\dot{P}_j = P_{\mathrm{a}} + p_j \mathrm{Cos}(\omega t + \theta_j) \tag{7}$$

Using Eqs. (2), (4), and (5), the pressure component of exergy at any location can be written as