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On controlling current distribution in superconducting cables

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Abstract

We present a revised version of a polynomial system modelling current distribution in a superconducting power cable. We show that by using the eigenvalue theorem in Algebraic Geometry, a numerical method can be developed to design a superconducting cable satisfying a predetermined current distribution.

Keywords: Homogeneous current distribution; Superconducting cables; Winding direction; Winding pitch

1. Introduction

There is a widespread belief that superconductivity is going to be a vital 21st century technology, not just in the power applications field but also in electronics. It is reflected on the extensive literature reporting on modelling, simulation and testing of superconducting devices. In particular, for electric power applications see the review in [7].

The interest in applying superconductivity to electric power and energy storage applications, is directly related to expectations for improved performance and efficiency advantages over conventional devices. In the case of superconducting cables, attractive is the larger amount of current and energy that can be transferred using superconductors compared to copper cables, and the energy savings that can be obtained with the superconductor. The superconducting cable is the object of study in this work.

Let us describe the device. A high temperature superconductor (HTS) power transmission cable is usually made of several layers of helically wound superconductor tapes. The current distribution among the conductor tapes is controlled mainly by pitches and winding directions of the layers, because the inductance of the layer is determined by the pitch and the winding direction.

As quoted in [9], *One of the most serious problems of this multi-layer alignment is non-uniform current distribution among the layers. If the layers do not share the current evenly, current capacity of the whole cable is much less than expected by critical currents of the conductors and the number of conductors.* Thus, the point of research, is to find efficient configurations, pitches and winding directions of the layers in order to satisfy the homogenous current condition. A relevant step in that direction is to propose and analyze mathematical models associated to the problem.

In [6], a generalized equation of the current distribution under the external field is introduced, and consequently the current distribution equation under a self-field only is derived. Therein all physical aspects are fully explained, we shall focus on the mathematical and numerical aspects of the model.

For a cable of m layers the following system is obtained:

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k} I_i + \left(\frac{\varepsilon_k}{L_k}-\frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}I_i\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2-\frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}I_i\right)=0,$$
$$k=1,2,\ldots,m-1. \tag{1}$$

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The parameters are:

- I_i: current in layer i.
- r_i: radius of layer i.
- ε_i: winding direction in layer i. $\varepsilon_i = -1, +1$ depending on the twist direction in the layer, Z or S.
- L_i: twist pitch in layer i.

A cable configuration is a set of values for ε_i, L_i. If a configuration and the radii are given, we can determine the current distribution in the cable combining the total current condition as follows:

$$\sum_{i=1}^{m} I_i = I_{\mathrm{T}}. \tag{2}$$

For a desired total transport current I_{T}, we have a linear system for currents I_i. To obtain physical solutions, parameters are to be chosen appropriately.

The problem of homogeneous current distribution can be formulated as follows: Assume that a cable of m layers is to be built with known radii r_i, $i = 1, 2, \ldots, m$. Find configurations satisfying (1) with $I_i = I_{\mathrm{T}}/m$, $i = 1, 2, \ldots, m$.

Hence the interest is to solve the system

$$k\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)} + \left(\frac{\varepsilon_k}{L_k} - \frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2 - \frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}\right) = 0,$$
$$k = 1, 2, \ldots, m-1, \tag{3}$$

for ε_i, L_i, $i = 1, 2, \ldots, m$.

We see that system (3) is quadratic on the variables ε_i/L_i. Also, it is known that in actual cables the number of layers is small, four or six are the most common. Thus, symbolic algorithms for solving polynomial systems are suitable for solutions. Some of these algorithms are based on techniques from Algebraic Geometry. In this work, we use the eigenvalue theorem in this theory to develop a hybrid algorithm, symbolic-numeric, to design a superconducting cable satisfying a predetermined current distribution. The algorithm rests on the theory of Gröbner bases. The content is as follows.

In Section 2 we present a slight modification of system (3) for modeling current distribution. A general algorithm for solving polynomial systems is sketched in Section 3. Also, the algorithm to solve the system of current distribution is presented. In Section 4 we provide some numerical examples of the applicability of the algorithm. More importantly, we report on actual cables whose current distributions correspond to the ones predicted by the algorithm. In Section 5 we comment on our work and future research.

2. Current distribution in superconducting cables

Instead of uniform current distribution we consider a more general situation. In normalized form, Eq. (2) reads

$$\sum_{i=1}^{m} I_i = 1. \tag{4}$$

In actual situations perfect efficiency is seldomly attained. Thus we introduce the efficiency equation

$$\sum_{i=1}^{m} E_i = E, \tag{5}$$

where $0 < E \leqslant 1$. Here E_i denotes the efficiency in layer i, $0 < E_i \leqslant 1/m$.

System (3) becomes

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + \left(\frac{\varepsilon_k}{L_k} - \frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}E_i\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2 - \frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}E_i\right) = 0,$$
$$k = 1, 2, \ldots, m-1. \tag{6}$$

Observe that in (6) the winding direction ε_i, and twist pitch L_i appear in the form ε_i/L_i. Letting $l_i = \varepsilon_i/L_i$ we obtain a quadratic system of $m-1$ equations with the m unknowns $l_1, l_2, \ldots, l_m$. Namely

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + (l_k - l_{k+1})\sum_{i=1}^{k}\pi r_i(l_iE_i) + \cdots + (l_k\pi r_k^2 - l_{k+1}\pi r_{k+1}^2)\sum_{i=k+1}^{m}(l_iE_i) = 0,$$
$$k = 1, 2, \ldots, m-1. \tag{7}$$

In the sections that follow, we shall present an algorithm to solve this system.

Remarks

(i) There are classical numerical techniques available to solve this polynomial system, but as quoted in [11], *there are no good, general solvers for solving systems of multivariate polynomial equations*. For systems of moderate size, symbolic algorithms ought to be considered. These algorithms reduce the problem to compute roots of a univariate problem, or to solve eigenvalue problems.

(ii) There is a great deal of experimental work on HTS cables. In [2] a report is presented on cables of 4, 8, and 10 layers which were built and tested. In [10], experiments were carried out in cables of 2, 4 and 10 layers also. In both cases current distribution is not uniform. The condition of (almost) homogeneous distribution is satisfied in cables reported in [12] (four layers) and [8] (six layers).