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Cryogenics 46 (2006) 699–704
Cryogenics
www.elsevier.com/locate/cryogenics

Performance comparison of thermoacoustic engines with constant-diameter resonant tube and tapered resonant tube

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Received 22 September 2005; received in revised form 21 April 2006; accepted 24 April 2006

Abstract

Two standing-wave thermoacoustic engines with a constant-diameter resonant tube and a tapered one, respectively, are simulated with linear thermoacoustics to explore the reasons for performance improvement of the thermoacoustic engine with the tapered resonant tube substituting for the constant-diameter one. Computed results indicate that the viscous loss in the tapered resonant tube is much lower than that in the constant-diameter one, and the smooth joint between the tapered resonant tube and its resonant cavity may avoid the acoustic power loss derived from sharp variation of flow area. The comparison between the computed results and the experimental data indicates that the simulation can roughly predict the performance of thermoacoustic engines with these two types of resonant tubes.

Keywords: Thermoacoustics (C); Thermoacoustic engine; Resonant tube; Tapered tube

1. Introduction

A pulse tube refrigerator driven by a thermoacoustic engine, instead of a mechanical compressor, is a potential long-life cryocooler due to no moving components from ambient to cryogenic temperatures [1–3]. A number of researches have been performed to improve the performance of thermoacoustic engines in recent years [4–12]. The researches indicate that the relative low pressure ratio is one of pivotal limits to the substitution of thermoacoustic engines for mechanical compressors as drivers for pulse tube refrigerators.

A resonant tube is of great importance for a thermoacoustic engine, since working frequency and profiles of pressure and velocity are significantly influenced by its geometries and dimensions [10]. Recently, tapered tubes, substituting for constant-diameter tubes, were used as resonators to enhance the gas oscillation in thermoacoustic engines [11,12]. With almost identical working frequencies, the replacement of the constant-diameter resonant tube (CDRT) by the tapered resonant tube (TRT) resulted in an increase of pressure ratio from 1.11 to 1.13 for a standing-wave thermoacoustic engine [11], and from 1.2 to 1.3 for a traveling-wave one [12]. These results attracted extensive interests, because the thermoacoustically driven pulse tube refrigeration in 60 K temperature region became possible with such a higher pressure ratio. References [11,12] gave a qualitative explanation for the enhancement with TRT from the nonlinear term $u\frac{\partial u}{\partial x}$ of a one-dimensional, inviscous momentum equation for a compressible fluid. Here, u is oscillating velocity of gas parcel and x stands for axial position. They pointed out that the TRT with suitable dimensions could reduce $\frac{\partial u}{\partial x}$, and then decrease the nonlinear term of $u\frac{\partial u}{\partial x}$. Thus, the nonlinear effect in high pressure ratio operations could be effectively depressed by TRT, and the acoustic energy could be focused on the fundamental-frequency mode. Consequently, the oscillation in the thermoacoustic engine could be enhanced with TRT.

According to the dimensions of the experimental setups reported in Reference [11], two standing-wave thermoacoustic engines consisting of a CDRT and a TRT, respectively,

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doi:10.1016/j.cryogenics.2006.04.006

are simulated with linear thermoacoustics in this paper. Based on the simulation, the acoustic power generation in stack and the acoustic power losses in the components of thermoacoustic engines are quantitatively analyzed, followed by a discussion of possible reasons for the oscillation enhancement with TRT substituting for CDRT.

2. Simulation of thermoacoustic engines with two types of resonant tubes

According to linear thermoacoustics [13,14], the momentum, continuity and energy equations for a short channel are as follows:

$$\frac{\mathrm{d}p_1}{\mathrm{d}x} = -(\mathrm{i}\omega l + r_\nu)U_1 \tag{1}$$

$$\frac{\mathrm{d}U_1}{\mathrm{d}x} = -\left(\mathrm{i}\omega c + \frac{1}{r_\kappa}\right)p_1 + eU_1 \tag{2}$$

$$\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x} = \frac{\dot{H}_2 - \frac{1}{2}\mathrm{Re}\left[p_1\widetilde{U}_1\left(1 - \frac{f_\kappa - \tilde{f}_\nu}{(1+\xi)(1+Pr)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_\mathrm{m} c_p |U_1|^2}{2A\omega(1-Pr)|1-f_\nu|^2}\mathrm{Im}\left(\tilde{f}_\nu + \frac{(f_\kappa - \tilde{f}_\nu)(1+\xi f_\nu/f_\kappa)}{(1+\xi)(1+Pr)}\right) - (AK + A_\mathrm{s}K_\mathrm{s})} \tag{3}$$

$$l = \frac{\rho_\mathrm{m}}{A}\frac{1-\mathrm{Re}(f_\nu)}{|1-f_\nu|^2} \tag{4}$$

$$c = \frac{A}{\gamma p_\mathrm{m}}\left[1 + (\gamma-1)\mathrm{Re}\left(\frac{f_\kappa}{1+\xi}\right)\right] \tag{5}$$

$$r_\nu = \frac{\omega\rho_\mathrm{m}}{A}\frac{\mathrm{Im}[-f_\nu]}{|1-f_\nu|^2} \tag{6}$$

$$r_\kappa = \frac{\gamma}{\gamma-1}\frac{p_\mathrm{m}}{\omega A\mathrm{Im}(-f_\kappa/(1+\xi))} \tag{7}$$

$$e = \frac{f_\kappa - f_\nu}{(1-f_\nu)(1-Pr)(1+\xi)}\frac{1}{T_\mathrm{m}}\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x} \tag{8}$$

where p_1 and U_1 are pressure amplitude and volume velocity amplitude, ω is angular frequency, ρ_m, T_m, c_p, γ, k and Pr are mean density, temperature, isobaric specific heat, specific heat ratio, thermal conductivity and Prandtl number of working fluid, respectively, f_ν and f_κ are viscous function and thermal function [14], A is flow area of channel, A_solid and k_solid are cross section area and thermal conductivity of the solid forming the channel, $\dot{H}_2$ is total power, ξ is a quantity presenting the effect of specific heat and thermal conductivity of the channel solid on the heat transfer between the working fluid and the channel (ξ equals to zero for the ideal solid with infinite specific heat and thermal conductivity.) [13], i is imaginary unit, Re, Im and superscript $\sim$ mean the real part, the imaginary part and the conjugation of a complex quantity. l, c, r_ν, r_κ, and e are inertance, compliance, viscous resistance, thermal-relaxation resistance and proportionality coefficient of a controlled source, which can be calculated with Eqs. (4)–(8).

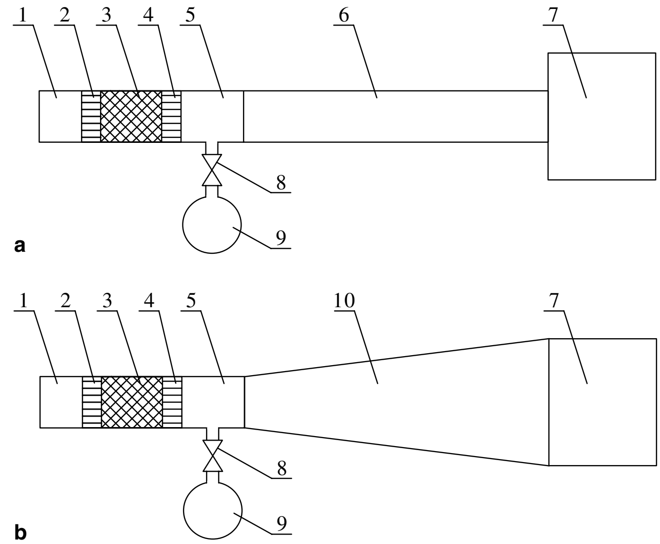

Fig. 1. Schematic of thermoacoustic engines: 1. hot tube, 2. heater, 3. stack, 4. water cooler, 5. T-shaped tube, 6. constant-diameter resonant tube (CDRT), 7. resonant cavity, 8. needle valve, 9. reservoir and 10. tapered resonant tube (TRT). (a) thermoacoustic engine with CDRT and (b) thermoacoustic engine with TRT.

The standing-wave thermoacoustic engines reported in Ref. [11] are shown in Fig. 1. It is seen that the two engines employ same hot tube, thermoacoustic core (composed of heater, stack and water cooler) and T-shaped tube, while geometry and dimensions of the resonant tubes and the resonant cavities are different. One consists of a TRT of 3 m in length with 0.08 m and 0.2 m diameters of two ends, and the diameter and the length of the resonant cavity are 0.2 m and 0.5 m, respectively. The working frequency of the engine with TRT was measured about 131 Hz in the experiments [11]. In order to compare the performance under identical working frequencies, the dimensions of the CDRT are chosen as 0.08 m in diameter and 2.21 m in length with a resonant cavity of 0.15 m in diameter and 0.345 m in length [11]. The main dimensions of the engines are tabulated in Table 1. Additionally, an RC (resistance and compliance) load consisting of a needle valve and a reservoir in series is connected to the engine to measure the acoustic power output. The volume of the reservoir is 1 l.

Table 1
Dimensions of thermoacoustic engines with CDRT and TRT

	Hot tube	Heater	Stack	Water cooler	T-shaped tube	CDRT and cavity		TRT and cavity	
						CDRT	Cavity	TRT	Cavity
Diameter (m)	0.08	0.08	0.08	0.08	0.08	0.08	0.15	0.08 → 0.2	0.2
Length (m)	0.15	0.05	0.134	0.039	0.473	2.21	0.345	3	0.5