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Effect of property variations on the operation of phase separators for superfluid helium

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Abstract

Many low-gravity space missions have used porous plugs to keep He II in a dewar while venting the vaporized gas to space. The operational state of a He II phase separator is determined by the interplay of several competing physical processes. It appears that one of these phenomena has not been included in past published models of a separator. When He II flows through a porous material that sustains a thermal gradient, large pressure maxima and moderate temperature maxima occur in the profiles. The transition of the superfluid component to the normal component causes the maxima. The ratio of the density of the two components is a thermodynamic property and the transition between the components adjusts the ratio to the local value of the pressure and temperature. This can be a large effect with the pressure maxima reaching ten times the saturation pressure. We present a model for a He II phase separator, similar to previous models, but with the property variation process included. We derive formulas that show the conditions when the effect is significant and the extent of the changes from constant property models. The property variation phenomenon is modeled using a modified Green's function approach with step functions. It is iterative on the pressure, temperature and velocity profiles starting with the constant property profiles. This process generates a power series expansion of the pressure, temperature and velocity profiles. Two iterations are sufficient for nearly all applications. The predictions of the model are compared with reported measurements and the agreement is good.

Keywords: Superfluid helium; Fluid dynamics; He II systems; Space cryogenics

1. Introduction

A porous plug phase separator (PPPS) is a component of the venting system for a majority of space missions requiring cooling when the cryogen is He II. The operational state of a He II phase separator is determined by the interplay of several competing physical processes. In past models, the effect of property variations has been neglected. The effect is rarely needed for modeling flows of ordinary fluids because it is proportional to a temperature difference times temperature derivatives of the thermodynamic parameters. In general, this is a small correction to the constant property solutions. The analysis presented below shows that a mechanism peculiar to He II leads to an additional factor of L/d^2 where L is the length of the He II region and d is the average hydraulic diameter of a flow path. When d is less than a micron, this factor is sufficiently large so that the effect of property variations is comparable to other phenomena included in published models. The purpose here is to develop the formulas for the effects of property variation and to include them in a model for a phase separator. The theory also applies to He II flows that are used in nanotechnology to probe structures considerably smaller than 1 μm.

The effects of property variation are inherent in the equations of motion. The author and Allan Mord developed a general-purpose computer program for He II based on the one-dimensional area averaged equations [1,2]. We called it SUPERFLOW. The flow path is divided into cells of equal length and the thermodynamic properties of each cell are evaluated at the temperature T and pressure p in

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each cell. The computer program was intended to assist in the analysis and design of applications for space missions and for the Superconducting Supercollider. The collider had very long cooling loops and SUPERFLOW predicted large pressure maxima and moderate temperature maxima in these loops. In addition, the velocities are not constant for constant diameter tubes. In many of the loops the pressure reaches ten times the saturation value of the end-point temperatures, the average temperature along the profile is higher than the higher end-point and the velocity of the normal component is two orders of magnitude above the constant property solution. These effects had not been described previously in the literature. We published an analysis of the data and many graphs of the profiles of the variables under varying conditions: Snyder and Mord [2]. The author found a dimensionless number that scales the shapes of all the profiles [2]. Mills et al. [3] verified the effects experimentally. Mills used a porous material with a mean passage diameter of 0.27 μm for the flow path. The agreement with the computations is excellent over a wide range of conditions. These unexpected results prodded us to look for a physical explanation.

The author proposed that these effects are due to the transition of the superfluid component to the normal component as the superfluid flows up a thermal gradient [2]. Zilsel [4] appears to be the first to call attention to the phenomenon of superfluid to normal component transitions. He cautioned that the superfluid density ρ_s should be treated as a variable—not a constant—in most calculations of He II flows. His warning has been ignored for the most part and it is rare that any effort is made to include property variations in calculations.

We could modify SUPERFLOW so that it can compute the operation of a PPPS. At present, it does not contain provisions for capillary forces and a variable length of the superfluid flow path. However, we prefer to develop a model that leads to analytic formulas. The design process is simpler if it is clear how each parameter scales and how it affects the output. Computer programs such as SUPERFLOW are ill-sited to demonstrate physical processes. The methods presented below result in formulas for all the unknowns in terms of the adjustable inputs. The formulas also show how superfluid transitions interact with other phenomena in phase separator operation.

2. Analysis

A modification of the Green's function method is used to derive the formulas. In this approach the source causing the result is extended in space. The source is divided into cells and the effect of an individual cell on the output is found. This is the Green's function for that cell. In our case this is called the step function. The Green's functions are added together to obtain the effect of the entire source. The process of combining the Green's functions is a convolution. For our problem, the flow path is divided into cells. The source is the increments of p and T associated with each cell Δp and ΔT. The output are the profiles of p, T and the velocities.

The procedure is to use the equations of motion to connect Δp and ΔT to the changes in the profiles and thus, to obtain the step functions. The Green's functions, for this application, are steps Δv_n, Δv_c, $\Delta\nabla p$ and $\Delta\nabla T$ at the location of the Δp and ΔT steps. A convolution of these step functions leads to the desired profiles.

2.1. Conservation equations for a transition

The total density of He II, ρ, has a negligible dependence on the pressure p and the temperature T for the range of conditions under which phase separators are employed. It is treated as a constant for the model described here. Since $\rho = \rho_s + \rho_n$, where ρ_s and ρ_n are the densities of the superfluid and normal components, a transition of ρ_s to ρ_n conserves ρ and $\Delta\rho_n = -\Delta\rho_s$. Flow through a passage such as a porous material or a tube also conserves the momentum m. We can write the momentum as $m = \rho_s v_c + \rho v_n$ where $v_c = v_s - v_n$ is the relative velocity and v_s and v_n are the velocities of the superfluid and normal components. Conservation of m requires $\Delta(\rho v_n) = -\Delta(\rho_s v_c)$. Likewise, the energy flux is $e = \mu\rho_s v_c + h\rho v_n$ where μ is the specific chemical potential and h is the specific enthalpy. The energy flux is also conserved and we can write $\Delta(\rho h)v_n = -\Delta(\rho_s \mu v_c)$. Accordingly, a transition of ρ_s to ρ_n also entails a transfer of momentum and energy flux from the superfluid to the normal fluid without a change in m or e. Since ρ_s is a strong decreasing function of T and a very weak function of p, there is a transition of ρ_s to ρ_n as He II flows up a thermal gradient. Changes in p and T cause the transition.

2.2. The equations for step heights

The flow equations of He II are fourth order and the four natural variables are: v_n, v_c, p and μ. We usually prefer to have results in terms of T instead of μ. The thermodynamic identity, $\mathrm{d}T = (1/\rho s)\mathrm{d}p - (1/s)d\mu$, may be used to restate any formulas that contain μ. Here, s is the specific entropy. The transition phenomenon causes all the variables to change. If we divide the length of the flow path L into N equal cells, we can find the changes in the variables in terms of the changes Δp and ΔT associated with a particular cell. The three conservation equations discussed above can be solved for Δv_n and Δv_c in terms of changes in thermodynamic parameters such as $\Delta\rho_s$ and Δh. For example, $\Delta v_n = -v_c(\rho_s/\rho sT)\Delta\mu - v_n(1/sT)\Delta h$. Brooks and Donnelly have shown that all the thermodynamic parameters can be calculated from ρ_s [5]. For example, $\Delta\rho_s = \partial\rho_s/\partial T|_p\Delta T + \partial\rho_s/\partial p|_T\Delta p$ and $\Delta h = \partial h/\partial\rho_s|_p\partial\rho_s/\partial T|_p\Delta T + \partial h/\partial\rho_s|_T\partial\rho_s/\partial p|_T\Delta p$. The terms multiplying Δp are usually much smaller than those multiplying ΔT, but they are carried along because Δp can be large due to the pressure maxima. The equations for He II consist of the conservation equations stated above and