Journal of Physics and Chemistry of Solids 95 (2016) 19–23

Contents lists available at ScienceDirect

Journal of Physics and Chemistry of Solids

journal homepage: www.elsevier.com/locate/jpcs

Dynamical parametric instability of carbon nanotubes under axial harmonic excitation by nonlocal continuum theory

Yi-Ze Wang [a,*], Feng-Ming Li [b]

[a] Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, China
[b] College of Mechanical Engineering, Beijing University of Technology, Beijing 100124, China

ARTICLE INFO

Article history:
Received 15 February 2016
Received in revised form
24 March 2016
Accepted 27 March 2016
Available online 30 March 2016

Keywords:
Carbon nanotubes
Parametric excitation
Instability region
Nonlocal continuum theory

ABSTRACT

Structures under parametric load can be induced to the parametric instability in which the excitation frequency is located the instability region. In the present work, the parametric instability of double-walled carbon nanotubes is studied. The axial harmonic excitation is considered and the nonlocal continuum theory is applied. The critical equation is derived as the Mathieu form by the Galerkin's theory and the instability condition is presented with the Bolotin's method. Numerical calculations are performed and it can be seen that the van der Waals interaction can enhance the stability of double-walled nanotubes under the parametric excitation. The parametric instability becomes more obvious with the matrix stiffness decreasing and small scale coefficient increasing. The parametric instability is going to be more significant for higher mode numbers. For the nanosystem with the soft matrix and higher mode number, the small scale coefficient and the ratio of the length to the diameter have obvious influences on the starting point of the instability region.

1. Introduction

In recent years, due to the outstanding mechanical, electrical and chemical properties, a lot of attention has been paid on carbon nanotubes [1–4]. With the excellent characteristics for the high stiffness and strength, low density and remarkable electronic characteristics, carbon nanotubes have been found many important and interesting applications. Furthermore, because of the highly resonance frequency, diminished active masses and tolerable force constants, vibration and dynamical properties are the special features of carbon nanotubes. They can make the nano-electromechanical systems (NEMS) suitable for the multitude technologies, such as ultra fast sensor, actuators and signal processing components, etc. Then, the vibration and dynamical characteristics require the further comprehensive investigations.

In order to present the mechanical analysis of carbon nanotubes, people have employed two main methods, i.e. the atomistic simulations and elastic continuum models. The molecular dynamics (MD) is the most common approach for the atomistic simulations but it requires extensive computations and need quite a lot of time. Instead, the continuum model such as elastic beams has been applied by many researchers. However, the classical continuum model cannot present the small scale effects which become rather significant for the nanostructures. Different from the classical continuum model, the nonlocal elasticity theory introduced by Eringen [5,6] can illustrate the small scale effects and received much attention on the buckling [7–10], vibration [11–18] and wave propagation [19–26] behaviors of nanostructures.

It is known that different from the free vibration properties, the forced vibration of elastic structures can present some unique information. For example, elastic beams under the axial harmonic load may lead to the phenomenon named as the parametric instability. The axial harmonic excitation can change the stiffness of the elastic system and result in the parametric vibration and instable region which are the main features of the elastic dynamics. During the forced vibration, the parametric instability may occur at some excited frequency regions, which is an important issue concerned by the researchers and engineers.

Although some results on the free vibration of nanotubes have been reported, studies on dynamical properties of carbon nanotubes under axial harmonic load are very limited. Due to the parametric instability by the axial harmonic load [27,28], investigations on the forced vibration need more attention. The present work is mainly focused on the parametric vibration and instability of double-walled nanotubes. Different from the classical continuum theory, the governing equation of double-walled nanotubes under the parametric excitation is derived by the nonlocal model. Based on the Galerkin's method, the numerical calculations can be performed with the coupled Mathieu equations. On the other hand, instable regions are presented to show the dynamic properties of nanotubes.

* Corresponding author.
E-mail address: wangyz@bjtu.edu.cn (Y.-Z. Wang).

http://dx.doi.org/10.1016/j.jpcs.2016.03.012
0022-3697/

2. Basic equations and solutions

The double-walled carbon nanotubes embedded in the elastic matrix under the axial harmonic load (i.e $F\cos\Omega t$) are shown in Fig. 1. The parametric excitation is along the longitudinal direction with the harmonic frequency Ω. The double-walled nanotubes are composed of two nested individual tubes with the thickness t for of each one. The structure length is L with the hinged boundary conditions and the elastic stiffness of the matrix is k_w.

According to the nonlocal continuum theory which considers the scale effects, it assumes that the stress at a reference point $\mathbf{x}$ in an elastic solid is a function of the strain at every point in the body. The basic equations can be given as [5,6]

$$\sigma_{kl,\,k} - \rho\,\ddot{u}_l = 0, \tag{1a}$$

$$\sigma_{kl}(\mathbf{x}) = \int_V \alpha(\mathbf{x}, \mathbf{x}')\tau_{kl}(\mathbf{x}')dV(\mathbf{x}'), \tag{1b}$$

$$\epsilon_{kl} = \frac{1}{2}(u_{k,\,l} + u_{l,\,k}), \tag{1c}$$

where σ_{kl} is the nonlocal stress tensor, ε_{kl} the strain tensor, ρ the mass density, u_l the displacement vector, $\tau_{kl}(\mathbf{x}')$ the classical (i.e. local) stress tensor, $\alpha(\mathbf{x}, \mathbf{x}')$ the kernel function which describes the influence of the strains at various location $\mathbf{x}'$ on the stress at a given location $\mathbf{x}$ and V the entire body.

The nonlocal constitutive relation of the one-dimensional stress state can be given as

$$\sigma_x - (e_0 a)^2\frac{\partial^2\sigma_x}{\partial x^2} = E\epsilon_x, \tag{2}$$

where E is the Young's modulus, e_0 the constant appropriate to each material and a the internal characteristic length (e.g. the length of C–C bond, the lattice spacing and the granular distance). Furthermore, e_0 is determined from experiments or by matching dispersion curves of the plane waves with the atomic lattice dynamics and e_0a denotes the scale coefficient which can describe the small scale effect of the nanostructures. If $e_0a=0$, it denotes the classical local model and e_0a is usually smaller than 2.0 nm from previous discussions.

For the Euler–Bernoulli beam model, the axial force and the resultant bending moment are

$$N = \int_A \sigma_x d\,A, \qquad M = \int_A z\sigma_x d\,A, \tag{3}$$

The vibration equation which is perpendicular to the x axis can be expressed as [29,30]

$$\frac{\partial S_1}{\partial x} = \rho A_1\frac{\partial^2 w_1}{\partial t^2} + c(w_1 - w_2) + F_1\cos\Omega\,t\frac{\partial^2 w_1}{\partial x^2}, \tag{4a}$$

$$\frac{\partial S_2}{\partial x} = \rho A_2\frac{\partial^2 w_2}{\partial t^2} + k_w w_2 + c(w_2 - w_1) + F_2\cos\Omega\,t\frac{\partial^2 w_2}{\partial x^2}, \tag{4b}$$

where $S_j = \partial M_j/\partial x$ is the shear force of the j–th nanotube with $j=1$ and 2, c the van der Waals interaction coefficient denoting the displacement coupling between the inner and outer nanotubes, the axial excitations $F_j=\sigma A_j$ and σ the axial stress load.

Then the governing equation of the parametric vibration for the double-walled nanotubes can be expressed as

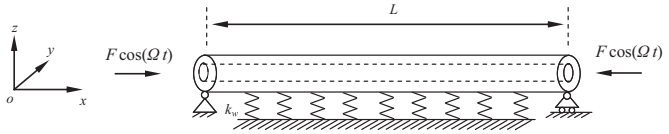

Fig. 1. Double-walled carbon nanotubes under harmonic parametric excitation by the nonlocal model.

$$\begin{aligned} EI_1\frac{\partial^4 w_1}{\partial x^4} = {} & \rho A_1\frac{\partial^2}{\partial t^2}\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right]w_1 \\ & + F_1\cos\Omega\,t\frac{\partial^2}{\partial x^2}\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right]w_1 \\ & + c\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right](w_1 - w_2), \end{aligned} \tag{5a}$$

$$\begin{aligned} EI_2\frac{\partial^4 w_2}{\partial x^4} = {} & \rho A_2\frac{\partial^2}{\partial t^2}\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right]w_2 \\ & + F_2\cos\Omega\,t\frac{\partial^2}{\partial x^2}\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right]w_2 \\ & + k_w\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right]w_2 \\ & + c\left[(e_0a)^2\frac{\partial^2}{\partial x^2} - 1\right](w_2 - w_1). \end{aligned} \tag{5b}$$

Moreover, the displacements for the inner and outer nanotubes can be expressed as [12,13]

$$w_j(x, t) = \sum_{k=1}^{\infty} W_{jk}(t)\sin\frac{k\pi\,x}{L}, \tag{6}$$

where W_j is the real constant and k is the mode number.

According to Eqs. (5a), (5b), and (6) and the Galerkin's method, we can derive that

$$\ddot{W}_1(t) + (\omega_{10}^2 - f_1\cos\Omega\,t)W_1(t) - \frac{c}{\rho A_1}W_2(t) = 0, \tag{7a}$$

$$\ddot{W}_2(t) + (\omega_{20}^2 - \cos\Omega\,t)W_2(t) - \frac{c}{\rho A_2}W_1(t) = 0, \tag{7b}$$

where

$$\omega_{10} = \sqrt{\frac{1}{\rho A_1}\left[c + \frac{EI_1}{1 + (e_0a)^2\left(\frac{k\pi}{L}\right)^2}\left(\frac{k\pi}{L}\right)^4\right]}, \tag{8a}$$

$$\omega_{20} = \sqrt{\frac{1}{\rho A_2}\left[c + k_w + \frac{EI_2}{1 + (e_0a)^2\left(\frac{k\pi}{L}\right)^2}\left(\frac{k\pi}{L}\right)^4\right]}, \tag{8b}$$

$$f_j = \frac{1}{\rho A_j}\left(\frac{k\pi}{L}\right)^2 F_j. \tag{8c}$$

With the following non-dimensional forms

$$\tau_j = \omega_{j0}t, \quad W_j = r_j g_j, \quad r_j = \sqrt{\frac{I_j}{A_j}}, \quad \bar{\omega}_j = \frac{\Omega}{\omega_{j0}}, \tag{9}$$

Eqs. (7a) and (7b) can be derived by Mathieu form as