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Analysis and optimization of a micromixer with a modified Tesla structure

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ABSTRACT

A flow-analysis method using Navier–Stokes equations has been applied to a parametric study on a micromixer with a modified Tesla structure, and an optimization of this micromixer has been performed with a weighted-average surrogate model based on the PRESS-based-averaging method. The numerical solutions are validated with the available numerical and experimental results. The mixing performance and pressure-drop have been analyzed with two dimensionless parameters, i.e., the ratio of the diffuser gap to the channel width, θ, and the ratio of the curved gap to the channel width, ϕ, for a range of Reynolds numbers from 0.05 to 40. The shape of the microchannel is optimized at the Reynolds number of 40 with two objectives: the mixing index at the exit and the friction factor. The “naïve approach” has been applied to realize a single-objective optimization problem. The optimization results reveal that the mixing and pressure-drop characteristics are very sensitive to the geometric parameters. Sensitivity analysis reveals that in the vicinity of the optimum point, the objective function is more sensitive to ϕ as compared to θ.

1. Introduction

With the trend of the miniaturization of various fluid systems, microfluidic devices, such as micropumps, microvalves, and micromixers, have been intensively researched. Microfluidic-mixing applications have expanded into many fields, including medical-drug delivery and biological, chemical, and thermal applications. The scaling down of fluid systems also raises some new issues, among which the micromixing of fluids is very important. Microfluidic devices have been widely utilized in ‘micro-total analysis systems’ (μTASs) or lab-on-a-chip systems for biological analysis, chemical synthesis, and clinical purposes. At microscopic scales, fluid mixing becomes very difficult. The dimension of a microfluidic device is typically in the order of sub-millimeters and conventional methods for stirring fluids are not applicable. More importantly, *Re* (the Reynolds number) is small, which means the flow is laminar and the mixing solely depends on molecular diffusion, which is usually very slow. Hence, in many applications, it is necessary to apply specially designed micromixers to promote mixing [1,2].

In order to enhance fluid mixing in microchannels, recently a variety of active and passive micromixers have been developed. Active micromixers depend on external fields to force fluids to mix together inside microchannels [3,4]. Active micromixers generally enhance mixing by stirring the flow in order to create secondary flows. This stirring-effect can be achieved by using additional structures or external sources, including ultrasonic vibration, dielectrophoresis, electrohydrodynamic, electroosmosis, and magnetic-force techniques. The secondary flow stretches and folds the interface of the fluids, thereby reducing the diffusion path between the fluid streams and increasing the mixing phenomenon. However, the fabrication of this type of microfluidic mixer is rather complex. Furthermore, these devices generally require some form of external power sources and control systems. Passive micromixers do not require external energy; the mixing process is governed by modifying the microchannel with different shapes or structures. Passive mixers can be further classified as lamination micromixers and injection micromixers. In lamination mixers, the fluid streams are divided into several small streams, which are later joined in a mixing channel [5]. On the other hand, an injection mixer splits only one stream into many sub-streams in the form of microplumes, which increase the contact surface and reduce the mixing path. In addition, active micromixers are generally more complex and thus, can be difficult to operate, fabricate, clean, and integrate into microfluidic systems. Passive mixers are used in most microfluidic applications. Bessoth et al. [6] reported a passive mixer that reduced the diffusion path between the fluid streams by splitting and recombining the flow.

Hong et al. [7] demonstrated an innovative, passive micromixer that uses the “Coanda effect,” which produces transverse dispersion with two-dimensional modified Tesla structures. The Coanda effect, named after Henri-Marie Coanda, who first identified the effect in 1910, involves the tendency of fluids to follow a surface; it can be the defining effect of a jet-flow phenomenon in which a jet attaches itself to a nearby surface and follows that curved surface, away from its initial direction. This structure makes use of the Coanda effect to split the fluid stream and to direct a part

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Nomenclature

H	curved gap distance (mm)
s	diffuser gap distance (mm)
B	depth of the channel cross-section
D	diffusion coefficient
Lc	axial length of the main channel (mm)
Lo	length of the channel inlet (mm)
Le	length of the channel outlet (mm)
W	width of the channel cross-section (mm)
N	number of sampling points
Re	Reynolds number
M	mixing index
c	mass fraction
F	objective function
w_f	weighting factor
f	friction factor
Δp	pressure-drop

Greek letters

μ	absolute viscosity of fluid (kg m^{-1} s^{-1})
ρ	fluid density (kg m^{-3}).
σ	variance
α, β	exponents for weighted-average models
θ	ratio of the diffuser gap to the channel width
ϕ	ratio of the curved gap to the channel width
λ	aspect ratio of the inlet cross-section

Subscripts

m	mixture
opt	optimum value
i	sampling point
max	maximum value
min	minimum value
x	axial distance (mm)

of the stream so that it recombines with the opposing flow of the other part of the stream. In this structure, the Coanda effect causes chaotic advection and significantly improves mixing. The physical properties of water were applied in the simulation. In the experiments, blue- and yellow-dyed DI (de-ionized) water were used as working fluids.

Asgar et al. [8] analyzed the mixing performance of a modified Tesla structure by both numerical and experimental investigations at a Reynolds number of 0.05. They experimentally observed 78% mixing at 5 mm downstream of the inlet in a 0.1 mm-wide microchannel. In the experiments of Asgar et al. [8], DI water that was fluorescently labeled with Dragon green dye and pure DI water were used as the working fluids. They also reported that experimental and numerical results showed good agreement with a 4% error on average. Hong et al. experimentally observed 72% mixing at a 7 mm distance from the inlet in a 0.2 mm-wide microchannel at $Re = 0.1$ and numerically reported that at the flow rate of 10 ml min^{-1} ($Re < 10$), a modified Tesla structure can achieve full mixing after fluid passes four mixing cell-pairs (which are within a length of approximately 7 mm). They also compared this result for the mixing with a T-type micromixer with the same cross-sectional area.

From the above discussion, it is clear that a modified Tesla structure is effective in enhancing the mixing of fluids by creating transverse dispersion. However, there have not been any reports on systematic investigations to find the effects of geometric parameters on the mixing performance and fluid-flow characteristics in a modified Tesla structure. However, for a staggered herringbone (grooved) micromixer (SHM, for short), Ansari and Kim [9] performed shape optimization by using the response surface method and a three-dimensional Navier–Stokes analysis of the flow. They reported that mixing can be effectively increased by optimizing the shape of the grooves.

In the present work, numerical analysis on mixing in a modified Tesla micromixer has been performed to investigate the variation of the mixing behavior and flow characteristics with geometric parameters for a wide range of the Reynolds number. Shape optimization has also been carried out with two objective functions: the mixing index at the exit and the friction factor. A weighted-average surrogate model is employed as a numerical optimization tool to obtain an optimal structure by considering two geometric design variables. Mixing in the channel has been analyzed through three-dimensional Navier–Stokes equations with two working fluids, viz., water and ethanol. The effects of two design parameters of a modified Tesla structure on the mixing behavior have been investigated at six Reynolds numbers that range from 0.05 to 40.

2. Numerical analysis

A schematic diagram of the modified Tesla geometry is shown in Fig. 1 with three units. The two different fluids, water and ethanol, enter from two inlets, as shown in the figure, and there is an outlet on the right side. The main Tesla channel is joined to the inlets at a T-joint. The width of the channel, W, is kept constant for all repeating units and the remaining part of the main channel. The dimensions are as follows: axial length of main channel (Lc) = 2.37 mm; width of the channel (W) = 0.2 mm; depth of the channel (B) = 0.2 mm; Lo = 0.1 mm; d = 0.175 mm; and Le = 2 mm. Le is the outlet channel length, starting form end of the Tesla units to the outlet. The other dimensions, h and s, vary from 0.03 mm to 0.07 mm and from 0.04 mm to 0.12 mm, respectively, while the total length of the channel is fixed at 4.47 mm, which is the sum of the channel-section lengths, i.e., Lo, Lc, and Le. The aspect ratio of the channel cross-section, W/B, is unity (the width of the channel is the same as the depth). The two inlets, Inlet 1 and Inlet 2, are merged in the main microchannel with a T-joint, as shown in Fig. 1. The properties of water and ethanol have been taken at 20 °C and are listed in Table 1. The diffusivity for both water and ethanol is 1.2×10^{-9} m^2 s^{-1}.

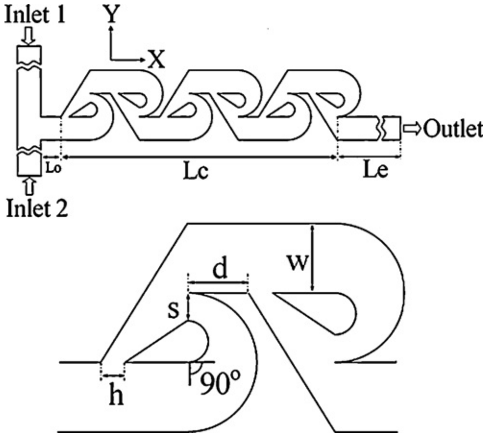

Fig. 1. Schematic diagrams of the modified Tesla structure.

Table 1
Properties of fluids at 20 °C.

Fluid	Density (kg m^{-3})	Viscosity (kg m^{-1} s^{-1})	Diffusivity (m^2 s^{-1})
Water	9.998×10^2	0.9×10^{-3}	1.2×10^{-9}
Ethanol	7.890×10^2	1.2×10^{-3}	1.2×10^{-9}