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# A general upper-bound formulation of stream function of upset forging of ring using a variational approach

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ABSTRACT

Based on an upper-bound energy dissipation equation established as a functional, a general formulation is derived for stream function of upset forging of ring using a variational approach. The general formulation consists of an equilibrium equation of the stream function along with a set of statically boundary conditions which were usually ignored in past investigation. In order to demonstrate the effect of the statically boundary condition on upper-bound solution and in order to verify the validity of the upper-bound solution as well, the experimental results also carried out in this investigation have been compared with the upper-bound solution for discussion. Both VUB and corresponding UB solutions were determined by optimizing a chosen stream function under consideration of the same degree of freedom. From the result we can clearly indicate that the VUB solution does present an improvement on the corresponding UB solution in predicting bulged profiles of upset forging of ring and disk. Such an improvement is only due to the fact that it is additionally imposed on the VUB solution the available statically boundary condition, which constrains plastic flow of the upsetting ring or disk on the frictional interface during upsetting process.



## 1. Introduction

In order to investigate the problem of upset forging of ring, a number of articles, based on the upper-bound theorem, have been published in the literature, e.g. [1–10]. These investigations are usually carried out in connection with proposing an effectual model of velocity field that is kinematically admissible only. The models proposed are usually expressed in terms of various functions, such as polynomial [1–3], trigonometric [4–6], and exponential functions [7,8], or a combination of these functions [9]. However, a few models, e.g., proposed by Nagpal [10], are expressed as an unknown function, from which different functions consisting of the polynomial, trigonometric, and exponential functions can be mathematically derived under certain considerations. Although the models proposed have been demonstrated to be applicable to analyze the problem of upset forging of ring or disk, they are all of kinematically admissible that can account for the constraint on given velocity condition at specific boundary only when employed to determine upper-bound solution. Meanwhile, the constraint on surface traction or force condition due to friction existing on interfacial frictional interface, which, we believe, would also affect metal flow in upset forging of ring or disk as experimentally observed in [1,2,11–14], has never been appropriately taken into consideration unless a variational approach is involved to derive it as reported in Refs. [15–17].

A method that determines an upper-bound solution using a variational approach is termed a variational upper-bound (VUB) method. It was initially proposed by Yeh and Yang [18] and has been successfully applied to the problem of cup ironing [18], plane strain extrusion [19,20] and upset forging of ring or disk [15–17]. In the forgoing cited articles where the VUB method was applied, an upper-bound energy dissipation equation has been established as a functional of the velocity field, assumed as an implicit function in prior, and then extremized using the technique of variational calculus in an attempt to derive an equilibrium equation along with a set of associated boundary condition of the velocity field. The derived boundary condition, which was termed as statically or naturally boundary condition and had never been accounted for in traditional upper-bound analyses such as in [1–10,21–25], was finally imposed to aim to determine upper-bound solution. The upper-bound solution determined in this way has been shortly termed the VUB solution, and has been shown to be better than the corresponding upper-bound solution that ignores the statically boundary condition in predicting the pattern of metal flow as well as the load required for a variety of metal forming problems [15–17,19,20]. Although the problem being considered has been discussed elsewhere [15–17] using a variational approach also, the

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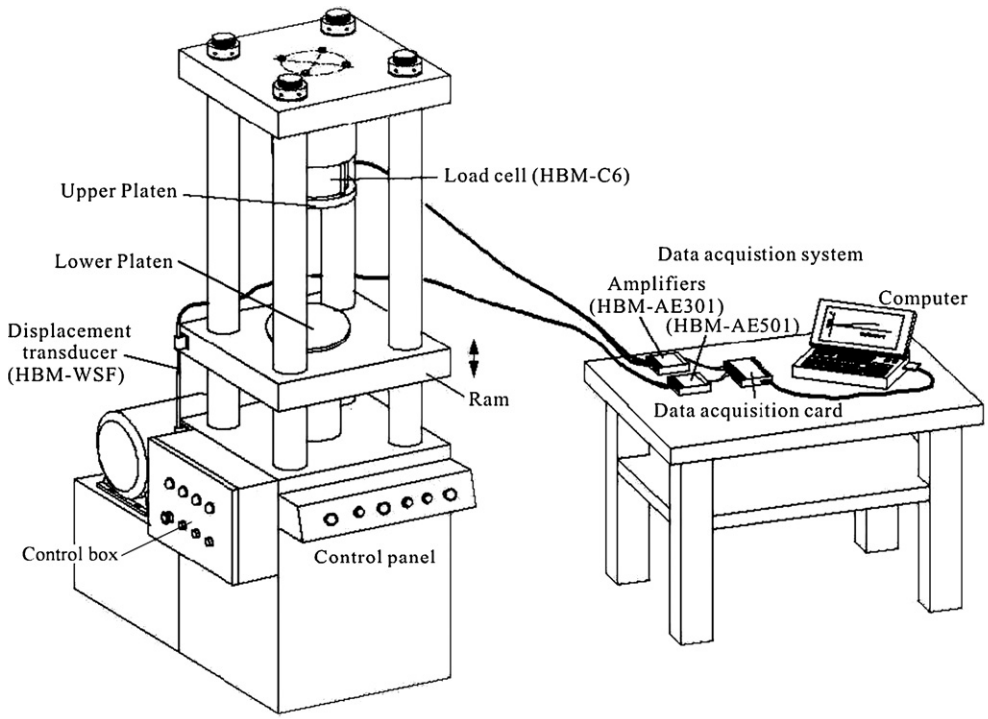


**Fig. 1.** Schematic illustration of 100 ton hydraulic testing machine.

major difference between the present and previous works is that the function used to establish energy dissipation equation is different physically. Specifically, in the previous works [15–17] the function, assumed implicitly in prior, was interpreted as velocity field; however, in the present work it is introduced to represent a stream function also assumed implicitly in prior, and thus any equation derived would be pertinent to the stream function itself rather than the velocity field. As the stream function may be known to be a potential of the velocity and/or strain-rate fields, it is expected that the formulation to be derived for the stream function would become more widespread in applications for the problem considered.

In this article, specimens made of Al6061 aluminum alloy were considered for experiments. One of the specimens was tested under uniaxial tension to determine the material constant of Al6061 aluminum alloy; the other was prepared for the test of upsetting in order to verify upper-bound solution. As a result of going through the theoretical derivation based on the upper-bound theorem, a general formulation of stream function consisting of an equlibrium equation along with a set of statically boundary condition can be obtained. Compared with the experimental result, upper-bound solutions determined with and without accounting for the statically boundary condition are presented and discussed for demonstrating effect of the statically boundary condition on the upper-bound solution. It is clearly seen from the result that the solution accounting for the statically boundary condition derived is considerably improved in the prediction of bulge profile of upset forging of ring and disk.

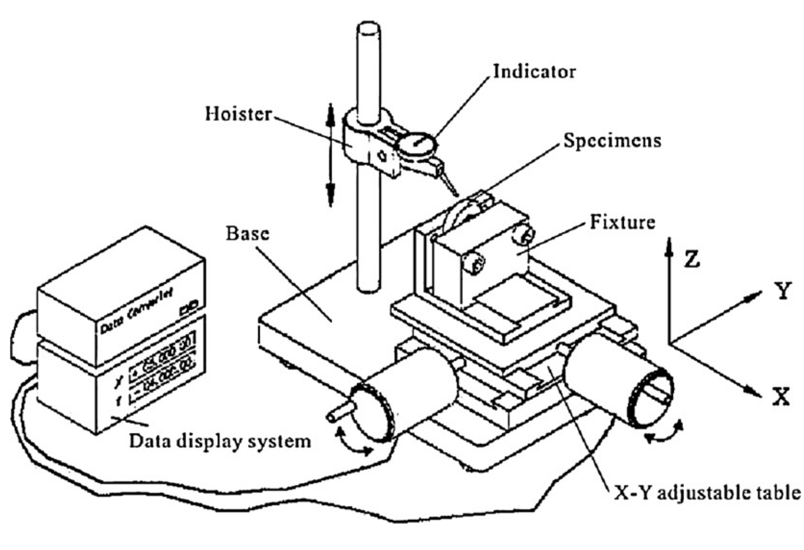


**Fig. 2.** Schematic illustration of digital measuring device.