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Short communication

Effect of dispersed phase rheology on the drag of single and of ensembles of fluid spheres at moderate Reynolds numbers

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ABSTRACT

In this work, the effect of dispersed phase rheology on the drag phenomena of single and of ensembles of fluid spheres translating in an immiscible power-law continuous phase has been studied numerically at moderate Reynolds numbers. The results presented herein encompass the following ranges of conditions: $1 \le Re_o \le 200$, $0.1 \le k \le 50$, $0.2 \le \Phi \le 0.6$, $0.6 \le n_i \le 1.6$ and $0.6 \le n_o \le 1.6$, thereby enabling the effects of the Reynolds number (Re_o), of the internal to external fluid characteristic viscosity ratio (k), of the volume fraction of the dispersed phase (Φ) and of the two power-law indices (n_i, n_o) on drag coefficient to be delineated. This information facilitates the estimation of the rate of sedimentation of single fluid spheres and their ensembles in quiescent continuous phase. Within the range of conditions studied herein, the effect of the dispersed phase rheology is found to be rather small.

1. Introduction

Due to their wide ranging applications in chemical, biological and processing industries, considerable effort has been devoted to the study of the hydrodynamic behaviour of fluid spheres in another immiscible liquid phase [1,2]. While numerous studies dealing with the behaviour of single particles provide useful insights, one frequently encounters ensembles of droplets in engineering applications. In recent years, considerable research efforts have been directed in developing reliable theoretical/numerical methods for the prediction of the settling velocity in liquid–liquid systems to evaluate their stability and/or to estimate the available contact time between the two phases. Over the years, significant information has been reported on the settling velocity of single fluid spheres [3–5] and their ensembles [6–8]. Thus, it is possible to estimate the settling velocity of the dispersed phase in these systems over conditions of practical interest when both phases are Newtonian fluids.

On the other hand, many high molecular weight polymers and their solutions, slurries, foams and emulsions encountered in several industrially important applications display shear-thinning, shear-thickening, yield stress and viscoelastic behaviour [9]. Due to the wide occurrence of non-Newtonian fluid behaviour, many studies are available which elucidate the influence of the continuous phase rheology (especially of shear-thinning and viscoelasticity) on the drag of single Newtonian fluid spheres, e.g., see Refs. [10–14] and for ensembles of fluid spheres [15–18]. Hence, reliable drag results are now available over the ranges of conditions ($1 \le Re_o \le 200$, $0.6 \le n_o \le 1.6$, $0.1 \le k \le 50$) for single spheres and over the range of volume fraction of the dispersed phase ($0.2 \le \Phi \le 0.6$) for ensembles. Broadly speaking, shear-thinning fluid behaviour reduces drag while shear-thickening enhances it as compared to its value in Newtonian continuous media otherwise under identical conditions.

In contrast, very little is known about the case when the dispersed phase or both phases exhibit non-Newtonian behaviour. For instance, Tripathi and Chhabra [19] used the velocity and stress variational principles to obtain approximate upper and lower bounds on the drag of a power-law fluid sphere falling in another power-law medium. The two bounds diverge with the increasing degree of shear-thinning behaviour, i.e., the decreasing value of the power-law index. Subsequently, this work was extended to obtain approximate upper and lower bounds on the drag (or rate of sedimentation) of ensembles of fluid spheres via the free surface cell model [20]. In the creeping flow region, both these studies suggested the influence of the dispersed phase rheology to be rather small in the limit of the zero Reynolds number. On the other hand, Gurkan [21] considered the case of a power-law drop falling in a

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Newtonian continuous phase. Their results embracing the range of conditions ($10 \le Re_o \le 50$, $0.1 \le k \le 1000$ and $0.6 \le n_i \le 1$) also suggest the effect of the dispersed phase rheology to be rather small, albeit their numerical results are believe be inaccurate in the limiting case of both phases being Newtonian fluids thereby casting some doubts about the reliability of their results for power-law fluids [3–5,13].

Similarly, there have been a few experimental results involving non-Newtonian fluid spheres settling in a Newtonian continuous medium. Marrucci et al. [22] reported experimental terminal velocity data in the creeping flow regime extending over the range of parameters $0.0045 \le k \le 1.88$ and $0.53 \le n_i \le 0.745$. However, due to the contamination by surface active agents which tend to immobilize the free surface of drops, their results are in complete agreement with the Stokes expression for solid spheres, even though the highest value of the viscosity ratio, k, in their study is only of the order of 2. Gillapsy and Hoffer [23] reported experiments on the drag coefficients of Newtonian and power-law liquid drops falling in air at large Reynolds numbers and reported no difference between the drag values for Newtonian and non-Newtonian liquid drops. This is also not at all surprising as their results relate to high values of the Reynolds number wherein the role of viscosity is expected to be small. Rodrigue and Blanchet [11] and Rodrigue [14] have carried out experimental studies on the motion and shapes of viscoelastic drops in another Newtonian and/or viscoelastic fluid with or without the presence of surfactants. However, the major thrust of their study was on shape transitions and thus no drag results were reported. Also, their experimental fluids exhibited both shear-thinning and viscoelastic characteristics and therefore, it is not possible to delineate the influence of these two characteristics.

It is thus clear that no prior results are available on the drag of single fluid spheres and their ensembles when the dispersed phase or both phases exhibit power-law fluid behaviour in the moderate Reynolds number range. This work aims to fill this gap in the literature.

2. Problem statement and description

Since extensive descriptions of the problems considered herein are available elsewhere [7,13,18], only the salient features are repeated here. A spherical coordinate system (r, θ, ϕ) with its origin at the centre of the drop is used with polar axis ($\theta = 0$) directed along the direction of flow. The flow is axisymmetric, i.e., v_ϕ is zero and no flow variable depends on the ϕ-coordinate. The dimensionless governing equations for this flow in their conservative form are:

- Continuity equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}[r^2(v_r)_{i,o}] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[(v_\theta)_{i,o}\sin\theta] = 0 \tag{1}$$

- r-component of momentum equation

$$\frac{\partial(v_r)_{i,o}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(v_r)^2_{i,o}] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[(v_r)_{i,o}(v_\theta)_{i,o}\sin\theta] - \frac{(v_\theta)^2_{i,o}}{r} = -\frac{\partial p_{i,o}}{\partial r} + \frac{2^{(n_{i,o}+1)}}{Re_{i,o}}\left[(\varepsilon_{rr})_{i,o}\frac{\partial\eta_{i,o}}{\partial r} + \frac{(\varepsilon_{r\theta})_{i,o}}{r}\frac{\partial\eta_{i,o}}{\partial\theta}\right] + \frac{2^{n_{i,o}}\eta_{i,o}}{Re_{i,o}}\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2(v_r)_{i,o}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial(v_r)_{i,o}}{\partial\theta}\right)\right] \tag{2}$$

- θ-component of momentum equation

$$\frac{\partial(v_\theta)_{i,o}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(v_r)_{i,o}(v_\theta)_{i,o}] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[(v_\theta)^2_{i,o}\sin\theta] + \frac{(v_r)_{i,o}(v_\theta)_{i,o}}{r} = -\frac{1}{r}\frac{\partial p_{i,o}}{\partial\theta} + \frac{2^{(n_{i,o}+1)}}{Re_{i,o}}\left[(\varepsilon_{r\theta})_{i,o}\frac{\partial\eta_{i,o}}{\partial r} + \frac{(\varepsilon_{\theta\theta})_{i,o}}{r}\frac{\partial\eta_{i,o}}{\partial\theta}\right] + \frac{2^{n_{i,o}}\eta_{i,o}}{Re_{i,o}}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial(v_\theta)_{i,o}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}[(v_\theta)_{i,o}\sin\theta]\right) + \frac{2}{r^2}\frac{\partial(v_r)_{i,o}}{\partial\theta}\right] \tag{3}$$

where subscripts i, o represent the internal (dispersed phase) and the external (continuous phase) flow variables, respectively. For an incompressible fluid, the extra stress tensor (τ_{xy}) is related to the rate of strain tensor (ε_{xy}) as:

$$\tau_{xy} = 2\eta\varepsilon_{xy}; \quad x,\ y = r,\ \theta,\ \phi \tag{4}$$

The viscosity of a power-law liquid is given as:

$$\eta = \left(\frac{\prod_\varepsilon}{2}\right)^{(n-1)/2} \tag{5}$$

where Π_ε is the second invariant of the rate of deformation tensor and its expression in terms of v_r and v_θ and their derivatives is available in standard books (e.g., see [24]). Eq. (5) represents shear-thinning, Newtonian and shear-thickening fluid behaviour for $n < 1$, $n = 1$ and $n > 1$, respectively. In the above equations, velocity has been scaled using U_o, radial coordinate using the drop radius R, pressure using ρU_o^2, components of the rate of strain tensor by U_o/R, viscosity by a reference viscosity η_{ref} ($= m(U_o/R)^{(n-1)}$), extra stress components by $\eta_{ref}(U_o/R)$ and time by R/U_o. Here m is the power-law fluid consistency index and n is the power-law behaviour index. The Reynolds number for the external phase is defined as follows:

$$Re_o = \frac{\rho_o U_o^{(2-n_o)}(2R)^{n_o}}{m_o} \tag{6}$$

The two Reynolds numbers, Re_i and Re_o are inter-related via the characteristic viscosity ratio and the density ratio as follows:

$$Re_i = \frac{Re_o\lambda}{k} \tag{7}$$

where ρ is the density of fluid, λ is the density ratio (ρ_i/ρ_o) and k is the characteristic viscosity ratio defined as:

$$k = \left(\frac{m_i}{m_o}\right)\left(\frac{2R}{U_o}\right)^{(n_o-n_i)} \tag{8}$$

For treating the motion of ensembles, within the framework of the free surface cell model [25], the inter-drop interactions are approximated by postulating each drop to be surrounded by a hypothetical envelope of the continuous fluid of radius R_∞ [1,7,8,15–18,25]. The dimensionless radius of the outer spherical envelope is related to the overall mean volume fraction of the dispersed phase, Φ, as:

$$R_\infty = \Phi^{-1/3} \tag{9}$$

Therefore, by simply varying the value of R_∞, one can simulate the ensembles of different volume fractions of the dispersed phase including the limiting case of a single droplet by setting $R_\infty \to \infty$, i.e., $\Phi \to 0$.

The relevant boundary conditions for this flow can be written in their dimensionless form as follows:

- At the outer boundary ($r = R_\infty$):

$$(v_r)_o = -\cos\theta \tag{10}$$