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Finite element modeling of the transient heat conduction between colliding particles

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Abstract

Finite element method (FEM) is employed to simulate the transient heat conduction during the collision between spherical particles. The total collision time is divided into many small time steps. At each time step, the contact area is evaluated by the Hertz's theory of elastic collision and based on this information, a grid system is generated for FEM computation to determine the temperature distribution in a particle and the heat exchange between particles. The total heat exchange is the sum of the heat exchange at all time steps. The FEM approach and computer code are verified by the good agreement between the numerical and analytical solutions for a well-established case. It is then used to simulate the transient heat transfer process during particle collision. It is shown that the heat exchange is affected by variables related to collision conditions and material properties. The results are qualitatively consistent with those obtained analytically based on the semi-infinite-media assumption. However, the analytical model overestimates the heat exchange, particularly when the Fourier number is high. A modified equation is proposed to overcome this problem based on the present FEM results. The equation is particularly suited for the newly developed particle scale modeling of the heat transfer of multiparticle systems.

Keywords: Finite element method; Heat conduction; Particle collision

1. Introduction

The transient heat conduction during the collision between particles is an important heat transfer mechanism in particle and multiphase systems, such as moving beds, fluidized beds and pneumatic conveying. Although this heat transfer mechanism may be neglected for dilute flow [1], it must be included for dense flow where particles are frequently in contact with their neighbors [2,3]. The study of this problem was pioneered by Soo [4]. He used the theory of elasticity to calculate the area and duration of contact between particles. In his analysis, the thermal conductivities of particles were assumed to be very high so that the temperature of a particle can be regarded uniform at any moment during a collision process. As a result, the resultant model is rather limited and only applicable for particles of large thermal conductivity. To overcome this problem, Sun and Chen [5] conducted both numerical and theoretical analysis of the transient heat conduction due to particle collision and proposed an equation for the calculation of the heat exchange between particles. In their study, the heat conduction between two particles was assumed to be similar to that between two semi-infinite media. This semi-infinite-media assumption is valid if the contact area between particles is very small compared to particle size. However, it may not truly represent the reality if the thermal conductivities of particles are large. More recently, Rong and Horio [6] conducted a numerical study to analyze the thermodynamic characteristics and NO_x emission of burning chars in a fluidized bed, where the particle–particle heat conduction is part of the model they developed. Their model involves various assumptions including the existence of gas layer between particles which could be opened for further investigation.

In recent years, development of a more general and accurate equation to calculate the heat transfer between particles becomes a significant issue, driven by the need to develop a

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Nomenclature

A^*	dimensionless contact area
A_c	maximum contact area (m^2)
A_f	contact area between two colliding particles (m^2)
c	specific heat (J/(kg K))
C	parameter in Eq. (19a)
C'	parameter in Eq. (20)
e	heat exchange given by Eq. (19a) (J)
e'	heat exchange given by Eq. (20) (J)
e_0	heat exchange given by Eq. (19b) (J)
E_i	elastic moduli of particle i (=1 or 2) (Pa)
E_{12}	defined by Eq. (3) (Pa)
Fo	Fourier number, $\alpha_1 t_c/r_c^2$
k	thermal conductivity (W/(m K))
KP	stiffness matrix
m_i	mass of particle i (=1 or 2) (kg)
m_{12}	defined by Eq. (4) (kg)
PM	mass matrix
r	radius coordinate (m)
r_c	maximum contact radius (m)
r_f	contact radius (m)
r_i	radius of particle i (=1 or 2) (m)
r_{12}	defined by Eq. (2) (m)
R	dimensionless radius coordinate
t	time (s)
t_c	time corresponding to r_c or A_c in a collision (s)
T	temperature (°C)
T_{i1}, T_{i2}	initial temperatures of particles 1 and 2, respectively (°C)
V	velocity (m/s)
z	axial coordinate (m)
Z	dimensionless axial coordinate

Greeks letters

α	thermal diffusivity (m^2/s)
ν_i	Poisson ratio of particle i (=1 or 2)
ρ	density (kg/m^3)
τ	dimensionless time, defined by Eq. (5) or (11)
φ	dimensionless temperature, defined by Eq. (11)

better description of heat transfer in particle systems and the connection with the newly developed simulation techniques [7]. For example, discrete particle simulation is now widely used to study the particle or particle-fluid flow at a particle scale (see [8,9] for example). The technique can also be used to study the heat transfer in such a flow system by properly incorporating the heat transfer between particles and structural information in the simulation, as demonstrated in recent studies [6,10–14]. Equations for heat conduction due to collision between particles are an integrated part in such microscopic studies.

This paper presents a numerical study of the conductive heat transfer between colliding particles by finite element method (FEM). It shows that the applicability of the semi-infinite-media assumption indeed depends on not only the contact area but also other physical parameters such as thermal diffusivity. Based on the present results, to facilitate particle scale modeling of heat transfer in particle systems, an equation is formulated to calculate the heat exchange between colliding particles.

2. Mathematical formulation

2.1. Elastic impact according to Hertz's theory

Consider two elastic smooth spheres of radii r_1 and r_2, elastic moduli E_1 and E_2, Poisson ratios ν_1 and ν_2, masses m_1 and m_2 are moving with a relative velocity V along the line of their centers when they collide. The two spheres are initially at different temperatures T_{i1} and T_{i2}, respectively. According to Hertz's theory of elastic collision, the change rate of the contact area A_f during this collision is given by [15,16]

$$\frac{dA_f}{dt} = \left[(\pi V r_{12})^2 - \frac{4}{5\sqrt{\pi}}\frac{E_{12}}{m_{12}}A_f^{5/2}\right]^{1/2} \tag{1}$$

where

$$r_{12} = \frac{r_1 r_2}{r_1 + r_2} \tag{2}$$

$$E_{12} = \frac{4/3}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2} \tag{3}$$

and

$$m_{12} = \frac{m_1 m_2}{m_1 + m_2} \tag{4}$$

Integrating Eq. (1) yields

$$\tau = \int_0^{A^*} \frac{dx}{(1-x^{5/2})^{1/2}} \tag{5}$$

where τ is the dimensionless time, defined by

$$\tau = \left(\frac{4E_{12}}{5m_{12}}\right)^{2/5} (r_{12}V)^{1/5} t \tag{6}$$

and A^* is the dimensionless contact area, defined by

$$A^* = \frac{A_f}{A_c} = \frac{r_f^2}{r_c^2} \tag{7}$$

where r_f is the contact radius at time t, A_c and r_c are the maximum contact area and the maximum contact radius, respectively. They are related, given by $A_c = \pi r_c^2$.A_c is calculated by [5,16]

$$A_c = \pi\left(\frac{5m_{12}{r_{12}}^2}{4E_{12}}\right)^{2/5} V^{4/5} \tag{8}$$

and its corresponding time t_c is given as

$$t_c = 2.94\left(\frac{5m_{12}}{4E_{12}}\right)^{2/5} (r_{12}V)^{-1/5} \tag{9}$$

According Eq. (5), the relation between the dimensionless contact area and the dimensionless time can be obtained.