Available online at www.sciencedirect.com
SciVerse ScienceDirect
Metamaterials
Metamaterials 6 (2012) 77–120
www.elsevier.com/locate/metmat

Review

Basics of averaging of the Maxwell equations for bulk materials

A. Chipouline [a,*], C. Simovski [b], S. Tretyakov [b]

[a] Institute of Applied Physics, Friedrich Schiller University Jena, Max-Wien-Platz 1, D-07743 Jena, Germany
[b] Department of Radio Science and Engineering, Aalto University, School of Electrical Engineering, PO Box 13000, FI-00076 Aalto, Finland

Received 8 August 2012; received in revised form 1 October 2012; accepted 12 October 2012

Abstract

Volume or statistical averaging of the microscopic Maxwell equations (MEs), i.e. transition from microscopic MEs to their macroscopic counterparts, is one of the main steps in electrodynamics of materials. In spite of the fundamental importance of the averaging procedure, it is quite rarely properly discussed in university courses and respective books; up to now there is no established consensus about how the averaging procedure has to be performed. In this paper we show that there are some basic principles for the averaging procedure (irrespective to what type of material is studied) which have to be satisfied. Any homogenization model has to be consistent with the basic principles. In case of absence of this correlation of a particular model with the basic principles the model could not be accepted as a credible one. Another goal of this paper is to establish the averaging procedure for bulk MM, which is rather close to the case of compound materials but should include magnetic response of the inclusions and their clusters. In the vast majority of cases the consideration of bulk materials means that we consider propagation of an electromagnetic wave far from the interfaces, where the eigenwave in the medium has been already formed and stabilized. In other words, in this paper we consider the possible eigenmodes, which could exist in the equivalent homogenized media, and the necessary math apparatus for an adequate description of these waves. It has to be again clearly emphasized, that the presented paper does not suggest new recipes for the homogenization procedure, but rather summarizes known basics in order to establish solid basis for more particular cases. Nevertheless, it is believed that any homogenization model has to be compatible with the presented in this paper general structure.

A discussion about boundary conditions and layered MM is a subject of separate publication and will be done elsewhere.

Keywords: Metamaterials; Homogenisation; Macroscopic averaging; Effective parameters

Contents

* Corresponding author.
E-mail address: arkadi.chipouline@uni-jena.de (A. Chipouline).

1873-1988/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.metmat.2012.11.001

1. Homogenization of Maxwell equations – macroscopic and microscopic approaches

1.1. Microscopic Maxwell equations and averaging procedure

We consider as a starting point a system of microscopic MEs in the following form:

$$\begin{cases} rot\,\vec{e} = \dfrac{i\omega}{c}\vec{h} \\ div\,\vec{h} = 0 \\ div\,\vec{e} = 4\pi\rho \\ rot\,\vec{h} = -\dfrac{i\omega}{c}\vec{e} + \dfrac{4\pi}{c}\vec{j} \end{cases} \quad \begin{cases} \rho = \sum_i q_i \delta\left(\vec{r} - \vec{r}_i\right) \\ \vec{j} = \sum_i \vec{v}_i q_i \delta\left(\vec{r} - \vec{r}_i\right) \\ \dfrac{d\vec{p}_i}{dt} = q_i\vec{e} + \dfrac{q_i}{c}\left[\vec{v}_i \times \vec{h}\right] \end{cases} \tag{1}$$

Here $\vec{e}$ and $\vec{h}$ are the microscopic electric and magnetic fields, respectively, ρ is the charge density, $\vec{q}_i$, $\vec{p}_i$, $\vec{r}_i$ and $\vec{v}_i$ are the charges, pulses, coordinates and velocities of charges, $\vec{j}$ is the microscopic current density, ω and c are the frequency and the velocity of light in vacuum. It is assumed that system (1) is strictly valid without any approximations. Actually, system (1) can be elaborated in the framework of the minimum action approach [31]; nevertheless, one should remember that the minimum action principle does not give an unambiguous form of the MEs (1), but instead gives a set of different forms which satisfy the requirement of relativistic invariance. The “right” form can be chosen based on the evident requirement of correspondence of the results of the final system of equations to the observed physical effects. One should also mention that in the framework of the minimum action approach the final equations are written for “potentials + particles”, not for “fields + particles”; the respective equations are:

$$\begin{cases} \dfrac{d\vec{p}_i}{dt} = -\dfrac{q}{c}\dfrac{\partial\vec{A}}{\partial t} - q_i\nabla\varphi + \dfrac{q_i}{c}\left[\vec{v}_i \times rot\,\vec{A}\right] \\ \dfrac{\partial F^{ik}}{\partial x^k} = -\dfrac{4\pi}{c}j^i,\ F_{ik} = \dfrac{\partial A_k}{\partial x_i} - \dfrac{\partial A_i}{\partial x_k} \end{cases} \tag{2}$$