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# Metamaterial properties of ferromagnetic antidot lattices

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**Abstract**

In this paper the metamaterial properties of two-dimensional arrays of circular antidots (holes) embedded into a ferromagnetic medium of Permalloy are studied according to both micromagnetic and analytical calculations. The periodicity of the arrays and the diameters of the antidots are in the nanometric range. The collective mode dynamics is described by means of effective physical quantities for the scattering geometry with the external magnetic field applied perpendicularly to the Bloch wave vector in the antidot plane. As an example, the definition of an effective field, incorporating the demagnetizing effects due to the holes, permits to describe the dynamical properties of collective modes in terms of effective properties in the travelling regime. An effective wavelength and a small wave vector are introduced both for extended and localized magnonic modes. By means of these effective quantities it is shown that holes play the role of point defects affecting the spin dynamics in the microwave range. Relations between the effective wavelength and the Bloch wavelength and between the corresponding small wave vector and the Bloch wave vector are found. Some effective rules on the dynamic magnetization, based upon the effective wavelength and the corresponding small wave vector, are derived. An application that exploits the definition of the small wave vector is proposed and an experiment based upon the notion of effective wavelength and small wave vector is suggested.


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## 1. Introduction

In these last years great attention has been devoted to the study of metamaterials both for their interesting properties and for the high number of potential applications. Among them the most studied are the electromagnetic metamaterials, because of their negative refractive index able to permit the creation of negative superlenses overcoming the diffraction limit [1,2]. Due to the high ratio between the vacuum electromagnetic wavelength and the periodicity of the structure, electromagnetic metamaterials [3] can be distinguished from the well-known photonic crystals [4–6]. An important step forward was done showing that it is possible to build a material exhibiting simultaneously negative effective permeability and negative effective permittivity in the microwave regime [7] confirming the prediction of Veselago [8]. In the last decade great efforts have been devoted also to the investigation of plasmonic metamaterials for their interesting optical properties. Since the wavelength of electromagnetic waves and of plasmonic waves is much larger than the periodicity of the artificial arrays, the description of the most important physical quantities can be made in terms of effective properties [9–11]. Other classes of widely investigated special

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materials that can be considered metamaterials are artificial dielectrics [12–14], materials with phononic waves [15] and chiral materials consisting of handed elements or handed microstructures [16].

On the other hand, the study of magnonic crystals, that represent another example of metamaterials, focusing on the propagation of spin waves that is modified by patterning [17] in periodic magnetic structures has received extensive attention. One of the first attempts to study the spin wave propagation in periodic systems was made by Elachi [18]. Recently, an active area of research devoted to the study of metamaterial properties of magnonic crystals is born [19–21]. A new class of metamaterials, the so-called magnonic metamaterials, are challenging not only for their fundamental properties, but also for their applications. In this respect, some studies on magnonic metamaterials exploiting the artificial periodicity have been carried out. The studied confined systems are of nanometric size. First, the effective electromagnetic properties of a medium containing ferromagnetic inclusions were studied theoretically [22]. A similar investigation was performed by taking as a magnetic material an antiferromagnet [23]. Secondly, a study of the effective properties of a one-dimensional (1D) magnonic crystal consisting of a periodically layered cylindrical nanowire was made and effective magnetic quantities were introduced [24]. In addition, investigations of the effective magnetic properties in a 1D array of rectangular dots [25] and in a two-dimensional (2D) array of magnetic nanoelements [26] were performed. Recently, also an analytical model based on the plane wave method able to determine successfully band structure and collective modes in real space for three-dimensional magnonic crystals was developed [27,28]. By using the partial wave method it is also possible to evaluate with great accuracy the effect of materials and of structural parameters on width of band gaps and to determine the effects on dynamics of any type of lattice and shape of scattering centres. Very recently, some metamaterial properties were obtained by studying spin dynamics in the long wavelength limit and by introducing an effective magnetization in 2D ferromagnetic arrays of circular antidots (ADs) [29]. Finally, opening of band gaps at the edges of Brillouin zones ($n$BZs) with $n = 1, 2, \ldots$ in a ferromagnetic 2D AD lattice have been investigated by analyzing the inhomogeneity of the internal field [30].

However, to the best of our knowledge, there are not yet in the literature studies of magnonic mode dynamics, free from restrictions in the reciprocal space investigation, focusing on the effective properties of 2D magnonic crystals. Indeed, the aim of this paper is to discuss the effective properties of a 2D periodic array of holes embedded into a Permalloy (Py) ferromagnetic medium. In order to do that, we investigate spin-wave dispersion in periodic systems of ADs both in the travelling and stationary regimes giving particular emphasis to the study of spatial profiles of the most relevant modes. Micromagnetic calculations were performed according to the dynamical matrix method (DMM), a finite-difference method based upon the solution of an eigenvalue/eigenvector problem where the matrix is expressed in terms of the second derivatives of the energy density evaluated at the equilibrium. Like the other finite-difference methods, the DMM presents several advantages: a single calculation yields the frequencies and eigenvectors of all modes of any symmetry and for any ground-state magnetization, it is applicable to a nanoparticle of any shape and the computation time is affordable. Hence, if compared to the analytical plane-wave method according to which the ground-state magnetization is always assumed uniform, the DMM permits to estimate with greater accuracy spin-wave mode frequencies. In this way, from the inspection of spatial profiles of magnonic modes at the high-symmetry points of BZs important effective properties can be extracted. The studied array periodicity and hole size are in the nanometric range. This description is made, according to micromagnetic and analytical calculations, by introducing effective quantities having a well defined physical meaning. The effective description adopted here for arrays of ADs is completely different from the one given in the case of electromagnetic and optical metamaterials where the electromagnetic radiation in the microwave region has a wavelength $\lambda$ that is much larger than the periodicity $a$ fulfilling the condition $\lambda \gg a$ [9]. Indeed, from micromagnetic calculations, it has been found that collective modes are characterized by an effective wavelength $\lambda_{\mathrm{eff}}$ that, compared to the hole size, fulfils the condition $\lambda_{\mathrm{eff}} \gg \delta$ with $\delta$ the hole diameter. In the studied system, holes play thus the role of point defects, but of finite dimensions affecting collective mode dynamics. The introduction of an effective wavelength allows us to define a corresponding small wave vector that has a different meaning with respect to the effective one introduced to explain the spin dynamics in 2D arrays of interacting circular dots [31].

The notions of effective wavelength and of the corresponding small wave vector can be exploited to build a magnonic device for obtaining a complete mapping of magnonic mode frequencies in the stationary regime. As it will be shown in Section 4, this could be achieved by applying a weak and periodic magnetic field and by knowing the only two values assumed by the wave