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Letter to the editor

On the history of backward electromagnetic waves in metamaterials

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Abstract

In this Letter to the Editor, I review briefly a historical background of the research on backward waves carried out in the XX century. I also provide a brief introduction to the isofrequency method of analysis for metamaterials and discuss a few unusual examples of wave phenomena at the interfaces.

Keywords: Metamaterials; Photonic crystals; Slow-wave structures; Backward waves; Negative refraction; Isofrequency analysis

1. Introduction

Metamaterials are often defined as the structures of metallic and/or dielectric elements, periodically arranged in three or two dimensions. Earlier, related structures were known as artificial dielectrics [1,2]. Electromagnetic wave propagation in such structures can be studied using an analogy with the de Broglie waves (see Table 1). This analogy is quite fruitful as it point to certain phenomena which were practically unnoticed in previous.

One should distinguish between the phenomena observed either in the stop-bands or in the pass-bands of periodic structures. When a single impurity is introduced to a periodic structure, local oscillation may occur at frequencies within the stop-band. It is analogous to the impurity energy levels in the band gap of a crystal, well-known in solid state physics. This results in a kind of resonator, so if a number of impurities are aligned sequentially, a transmission line emerges. Similarly, coupled transmission lines can be obtained. This way, local oscillations can be exploited for various passive devices, such as filters, splitters, directional couplers, *etc*. Furthermore, passive devices can be extended to the active ones by means of diodes or transistors. The above logic is used in photonic crystals; on this basis, amplifiers, generators and other applications were made [3] for optics as well as lower frequency range.

In recent years, a lot of interest emerged to the propagation of electromagnetic waves in the pass-bands of periodic structures. Particular attention is paid to the structures with the negative refractive index, occurring when phase and group velocities are oppositely directed. In that case, one speaks of *backward waves*.

2. On the history of backward waves

To the best of my knowledge, backward waves are first mentioned in the work [4] of Lord Rayleigh in 1877, where the relationship between the phase v and group v_g velocities was derived, which reads, in terms of delay coefficients $n = c/v$ and $n_g = c/v_g$, as

$$n_g = n - \lambda \frac{dn}{d\lambda},$$

where c is the light velocity and λ is the free-space wavelength.

1873-1988/$ – see front matter
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Table 1
Analogy of de Broglie and electromagnetic waves.

de Broglie waves	Electromagnetic waves
Electron energy $E = \omega$	Frequency ω
Quasi-momentum p	Wave vector β
Electron velocity $v_e = \nabla_p w$	Group velocity[a] $v_g = \nabla_\beta w$
Dispersion $E(p)$	Dispersion $\omega(\beta)$
Energy zone	Pass-band
Energy gap (forbidden zone)	Stop-band
Isoenergetic surface	Isofrequency
Impurity (surface) levels	Local oscillations

[a] Equals to the velocity of energy transfer.

The phase delay coefficient $n = c/v$ is essentially the same as refractive index. With $n > 0$ and $\mathrm{d}n/\mathrm{d}\lambda$ being sufficiently large, group delay can be negative, which means that the phase and group velocities are oppositely directed. There have been no examples of such media, however, until a mechanical analogy was provided by H. Lamb in 1904 [5], based on contracted string or compressed membrane. In the same year, Schuster [6] noticed that a plane wave incident from free-space onto a medium which support backward waves, is refracted in opposite direction with respect to normal, as compared to a usual case. This result was discussed in detail by Mandelstam [7,8] in his lectures in 1940. Furthermore, he attempted at experimental observation of this phenomenon, making use of the dispersion characteristics of NaCl-like lattices, where both acoustic and optical branches are present, with the backward waves possible for the optical branch.

In 1948, M. F. Stelmakh registered an invention [9] of a tube with prolonged electron-wave interaction, for generation in the wavelength range 10.2–11 cm. As judged by the operating voltage and delay system size, that was an example of backward-wave tube, with the opposite group and phase velocities, although this is not explicitly mentioned in the description.

In 1953, backward-wave tube was reported by Kompfner and Williams [10], and since then, a number of publications appeared on the periodic structures which support backward waves. In essence, the forward- and backward-wave tubes employ the Vavilov–Cherenkov effect, and in the backward ones, the reverse Vavilov–Cherenkov effect occurs, so that the energy is emitted opposite to the direction of the electron beam. Later on, backward waves were introduced in antenna technology [11].

Aiming at increase in the power of high-frequency generators and amplifiers, as well as providing technological platforms for still higher frequencies, attempts were ongoing since 1956 to use the 2D periodic structures in electronic devices [12]—see Ref. [13] by Doehler and co-authors. To illustrate the dispersion characteristics, isofrequency contours were used, showing the magnitudes of the wavevectors for a given frequencies. These contours are analogous to the isoenergy surfaces in solid state physics, and are also called wavevector surfaces in electrodynamics and optics. Isofreqency plots revealed structures which could support both forward and backward waves—many of such structures would have been called metamaterials these days. Quasi-optical properties of such structures, however, were not analysed in [13].

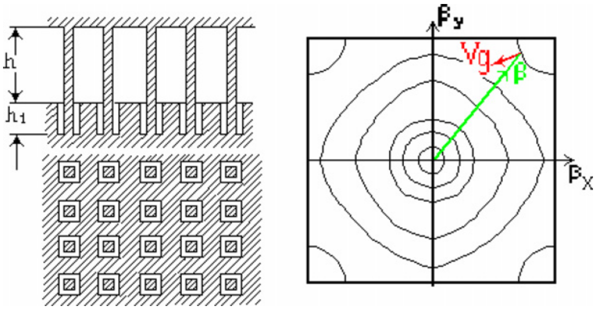

Fig. 1. An example of the media that has $n < 0$ in two dimensions in the first passband, showing the corresponding wave vector β and group velocity $\mathbf{v}_g$ on the isofrequency plot.

Back in 1958, the author was studying the dispersion characteristics of certain metamaterials (see Figs. 1 and 2), which support backward waves [14,15]. In the article [14], quasi-optical properties are also considered along with the isofrequency contours, and it is pointed out that backward-wave media provide negative refraction as predicted by Schuster and Mandelstam. This is further discussed in the book [19] published in 1966 (English translation, 1971). Waves in metamaterials were subsequently studied by the author up to 2000 [20–26].

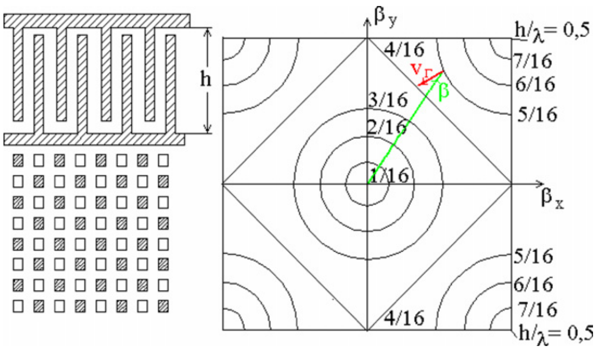

Fig. 2. 2D-periodic structure with the “interdigital” rods, and the corresponding isofrequency contours, showing wave vector β and group velocity $\mathbf{v}_g$.