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Metamaterials
Metamaterials xxx (2012) xxx–xxx
www.elsevier.com/locate/metmat

A novel approach to determine the plasma frequency for wire media

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Received 27 April 2012; received in revised form 17 July 2012; accepted 23 August 2012

Abstract

Analytical, simulation and experiment based technique has been reported in this paper to determine the plasma frequency of artificial media formed by arrays of parallel conducting wires called wire media. A generalized analytical approach based on quasi-static analysis has been done and compared with previously reported results to determine the plasma frequency of wire media. An eigenmode solver based simulation method has been used for plasma frequency extraction of infinite wire array using a commercial finite element method (FEM) based electromagnetic solver. The limitations of scattering-parameter based technique has been discussed and a new loss-factor method has been proposed. On the basis of simulated data, wire array has been fabricated and experiments has been carried out at X-band (8.2–12.4 GHz). Loss-factor method has been validated using the experimental data. Finally, the results of different techniques are compared to establish the efficacy of the loss-factor method supported by experimental results.

Keywords: Wire media; Plasma frequency; Metamaterial; Quasi-static approach; Unit cell; Eigenmode solution; Loss-factor method

1. Introduction

Metamaterials are of considerable interest to researchers today as they exhibit exotic electromagnetic properties like reversal of Snell's law, reversed Doppler effect and reversed Cherenkov radiation. Out of many structures proposed, periodic array of thin metallic wires has been used by various researchers to realize negative effective permittivity [1–6]. Wire media was known to behave like plasma at least since 1950s and Brown [1] proposed a close form formula for effective permittivity and cut-off wavelength i.e. plasma wavelength of rodded (wire) medium. Wire media has also been investigated by many researchers [7–9] for its unusual electromagnetic properties. An important parameter to characterize such media is the plasma frequency, from which the effective permittivity can be found out. Pendry et al. [10] showed that the effective relative permittivity of the wire media can be obtained in terms of the plasma frequency f_p as:

$$\epsilon_{reff} = 1 - \frac{{f_p}^2}{f^2 + 2i\gamma f} \tag{1}$$

Pendry's formulation also shows that f_p can be evaluated in terms of wire radius r_0 and lattice period a as:

$$ {f_p}^2 = \frac{c^2}{2\pi a^2\ \ln(a/r_0)} \tag{2}$$

where c is the speed of light in free space. Different analytical formulation for the plasma frequency has been

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http://dx.doi.org/10.1016/j.metmat.2012.08.001

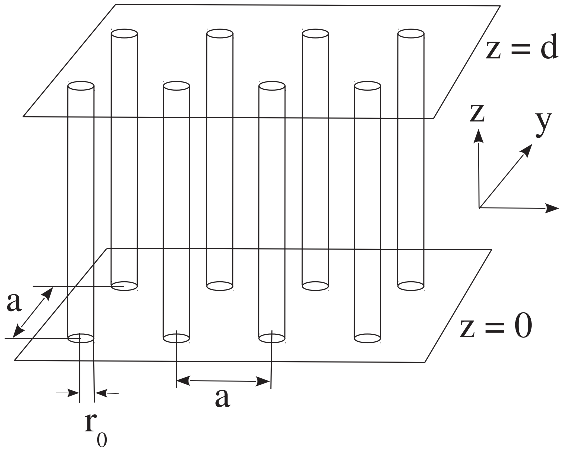

Fig. 1. Schematic of wire medium.

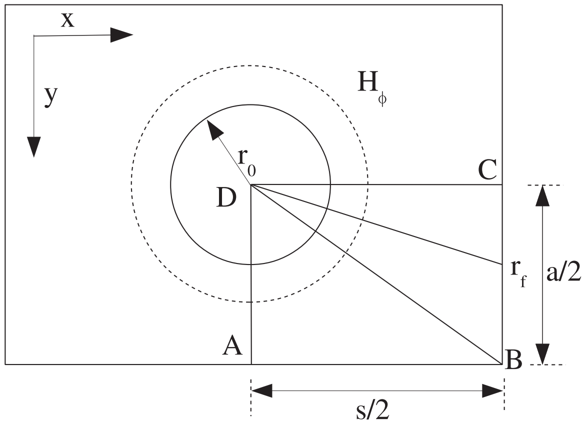

Fig. 2. Cross-section of rectangular unit cell of wire medium.

given by Sarychev et al. [11], Markos et al. [12], Belov et al. [13] and Maslovski et al. [14].

In this paper, a general model has been proposed for rectangular unit cell which can be reduced for square unit cell by equating the lattice constants in two directions using quasi-static approach [14]. The dependence of plasma frequency on the unit cell size has been established. Further, an eigenmode solver based technique has been discussed. These analytical or eigenmode solver based approaches consider an infinite wire array for analysis. So these approaches poses difficulty in experimental verification. The limitations of commonly used scattering-parameter based technique has also been discussed in this regard. To cope up with all these difficulties, a new approach called loss-factor method [15] is proposed. On the basis of simulation data an wire array has been fabricated at X-band (8.2–12.4 GHz) to compare all the proposed approaches.

2. Analytical estimation of plasma frequency

Maslovski et al. [14] developed a quasi-static model using square unit cell, by computing the total magnetic flux per unit length in the unit cell assuming that distribution of magnetic field is same for any radial direction and hence calculating magnetic field in one linear direction. However, magnetic field has to be considered in all radial directions to account for both square and rectangular unit cells. So we extend their model and developed a generalized quasi-static approach as discussed below.

The problem geometry is shown in Fig. 1. Two equipotential imaginary planes at $z = 0$ and $z = d$ are considered in an infinite wire medium that are orthogonal to the wires. An external electric field parallel to the z-axis has been applied to the wire medium such that, an electric current I_0 flows through each wire. If $d \ll \lambda$ then the medium may be treated as a parallel plate capacitor and the plasma frequency can be found out from quasi-static considerations.

The electric displacement vector $\overline{\mathrm{D}}$ and the electric field $\overline{\mathrm{E}}$ are related by the following equation [14]

$$\overline{\mathrm{D}} = \left(\epsilon_0 \bar{\mathrm{I}} - \frac{\widehat{z} \cdot \widehat{z}}{\omega^2 a^2 \mathbf{L}} \right) \cdot \overline{\mathrm{E}} \quad (3)$$

where $\overline{\mathrm{D}}$ $(= \epsilon_{eff} \overline{\mathrm{E}})$ and $\overline{\mathrm{E}}$ are average displacement vector and electric field in medium. $\bar{\mathrm{I}}$ is the unit matrix and $\widehat{z}$ is the unit vector in z direction. $\mathbf{L}$ is the wire inductance per unit length, and ω is the angular frequency. To find out the wire inductance per unit length, one needs to estimate the magnetic field distribution in the medium. The magnetic field can be estimated by considering a unit cell due to the symmetry and infinite periodicity of the medium. The generalized rectangular unit cell case has been considered first from which the case of square unit cell follows.

2.1. Rectangular unit cell

Fig. 2 shows the cross section of rectangular unit cell with the wire in the center. Due to symmetry, magnetic field is zero at the middle point between any two wires i.e. at unit cell boundaries. In the triangle BCD, the circumferential magnetic field between two wires can be written as

$$H_\phi(r, y) = \frac{I_0}{2\pi} \left(\frac{1}{r} - \frac{1}{2r_f - r} \right) \quad (4)$$

where

$$r_f{}^2 = \left(\frac{s}{2} \right)^2 + y^2 \quad (5)$$