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Magneto-inductive conductivity sensor

Timmy Floume*
Imperial College London, Optical and Semiconductor Devices Group, Department of Electrical and Electronic Engineering, Exhibition Road, London SW72BT, United Kingdom

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Abstract

Multi-element inductive coil systems are used to measure locally resolved conductivity profile. Usually such sensors rely on the separate interrogation of each coil. In addition, the coils must generally be magnetically decoupled for accurate signal processing. Here we demonstrate a metamaterial conductivity sensor that uses broadband interrogation of a line of coupled resonators. No decoupling is needed, which allows a transmission measurement to be carried out. The resonant elements of the metamaterial are coupled with each other and their neighbourhood which affects their quality factor. We derive analytically an algorithm to extract the local perturbation in each element from the modal measurement. We investigate numerically the performance of the sensor and derive an optimal configuration in terms of nearest neighbour coupling and the initial non-uniformity. Finally we implement a four-element magneto inductive conductivity sensor and show that a conductive perturbation along the line can be accurately reconstructed. Generalisation to higher number of elements is also discussed.

Keywords: Conductivity; Sensor; Q-factor; Magneto inductive waveguide; Metamaterial; Coupled system

1. Introduction

Metamaterials are often envisaged from a material perspective, as homogenisation principle gives them unorthodox electromagnetic properties [1,2]. However, at a smaller scale, each of the metamaterial element is sensible to its surroundings, and if the number of element finite to boundary conditions. We investigate here a simple RF model of metamaterial made of closely coupled electrical resonators. For such arrangement, dissipation is generally perceived as a limitation [3,4]. Losses in resonant element originate from the finite conductivity of the constitutive metallic structure and also from the dissipation of the electromagnetic field. In particular, when the resonant elements are made of air-cored inductors, as is the case with split ring resonators, the magnetic field is not confined to the metamaterial and can induce eddy currents in a conductive surrounding medium. Because the dissipation of those currents generates additional losses, a metamaterial is sensitive to the conductivity of its surroundings [5]. Here we investigate how this property can be used to build a spatially resolved conductivity sensor from a metamaterial line.

Inductive measurements are generally made with single coil. Examples include radio frequency identification and metal detection. When multi-coil assemblies are used, as for magneto inductance tomography for 3D mapping of tissue conductivity [6–10], the coupling between them is minimized as it interferes with the extraction of information from the individual readout of each coil. This forces the parallel processing of the induced signals. Here we question the necessity for

* Tel.: +44 02075946216.
E-mail address: t.floume@imperial.ac.uk

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such decoupling, as coupling allows the extraction of distributed information from a single localized measurement (i.e. a serial broadband measurement). We believe that the reduction in the number of channels and the lack of decoupling could be beneficial for some application (e.g. the measurement of patient loading in magnetic resonance imaging).

We consider a simple system consisting of an array of magnetically coupled resonators forming a magneto-inductive line over which we propagate a wave and extract a single broadband measurement. Each element interacts with its neighbouring elements via inductive coupling and with the medium via induction of eddy current. Our approach is similar to the general theory of magneto-inductive material [11–15]. For a finite line of resonators, boundary conditions result in standing waves (i.e. modes) in the array. We start by solving the direct problem of finding the modal quality (Q-) factors from the Q-factors of the individual elements. Using perturbation theory we derive a linear relationship between the two sets. Inversion of this relationship, if possible and accurate, should therefore enable the calculation of losses in each element from the measurements of the dissipation in characteristic modes.

We demonstrate that by optimising the properties of the line (i.e. the self resonant frequencies of the elements and the coupling between them), it is possible to extract with a good precision the resistive losses in each element from the broadband measurement of the line characteristic resonances.

2. Theory

We start by studying a lossless magneto-inductive (MI) line and show that it is equivalent to solving an eigenvalue equation [14]. Losses in each element represent a diagonal perturbation of the eigenvalue equation. We use perturbation theory [16] to demonstrate how measurements of changes in modal Q-factors can be processed to derive the loss in each element, and hence find the local conductivity.

2.1. Problem description

We consider a finite line of N magnetically coupled lossless L–C resonators (see schematic on Fig. 1a and b). The nth element is formed by an inductor L and a capacitor C_n so that its angular self-resonant frequency is $\omega_n = 1/\sqrt{(LC_n)}$. It is coupled to its nearest neighbours by mutual inductance M. Without excitation, the circuit equations may be found from Kirchnoff's voltage law as:

$$\left\{1-\left(\frac{\omega_n}{\omega}\right)^2\right\} I_n + \left(\frac{\kappa}{2}\right)\{I_{n-1}+I_{n+1}\} = 0 \qquad (1)$$

Here, ω is the angular frequency of the signal being propagated, I_n is the current in the nth element and $\kappa = 2\,M/L$ is the coupling coefficient. Assuming that the line is finite and extends from $n = 1$ to N, and currents are zero in hypothetical 0th and $N+1$th elements, we can write the N equations (1) in a matrix form:

$$\left\{\underline{K}-\left(\frac{\omega_0}{\omega}\right)^2\underline{T}\right\}\mathbf{I} = 0 \qquad (2)$$

Here, $\mathbf{I}$ is a vector containing the currents I_n, ω_0 is an arbitrary angular frequency, $\underline{T}$ is a diagonal matrix with elements value $(\omega_n/\omega_0)^2$ accounting for the non-uniformity of the self-resonant frequency of each element, and $\underline{K}$ is a tri-diagonal matrix with diagonal elements equal to 1 and lower and upper diagonal elements equal to $\kappa/2$, characterizing nearest-neighbour coupling. Throughout the manuscript, we use underlined capital letters to describe matrices and bold letters to describe vectors. We can rewrite Eq. (2) as an eigenvalue equation by pre multiplying by $\underline{T}^{-1}$ to get:

$$\left[\underline{T}^{-1}\underline{K}-\left(\frac{\omega_0}{\omega}\right)^2\right]\mathbf{I} = 0 \qquad (3)$$

We now consider the loss in each element (shown in Fig. 1a as R_{in} and R_{en} for the intrinsic losses in the track and the losses due to induced eddy current dissipation [17]). The frequency dependant loss term can be expressed as a function of the Q-factor, $Q_n = L\omega_n/(R_{\mathrm{in}} + R_{\mathrm{en}})$, of the nth element at resonance. Eq. (1) must therefore be rewritten as:

$$\left\{1-\left(\frac{\omega_n}{\omega}\right)^2 - j\left[\frac{\omega_n}{\omega Q_n}\right]\right\} I_n + \frac{\kappa}{2}\{I_{n-1}+I_{n+1}\} = 0 \qquad (4)$$

and it follows that Eq. (2) must be rewritten as:

$$\left[\underline{T}^{-1}\underline{K}'-\left(\frac{\omega_0}{\omega}\right)^2\right]\mathbf{I} = 0 \qquad (5)$$

where $\underline{K}'$ is a tri-diagonal matrix with diagonal elements equal to $1 - j[\omega_n/(\omega Q_n)]$ and upper and lower diagonal elements equal to $\kappa/2$. The new matrix $\underline{K}'$ is therefore the sum of the original matrix $\underline{K}$ relevant to the lossless case and a frequency dependant diagonal matrix $\Delta\underline{K}(\omega)$ with elements $-j[\omega_n/(\omega Q_n)]$.

We now note that we are interested in the solution of (5) at the resonant frequencies of the mode only. This has two important consequences. First, we can solve