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Homogenization of tensor TL metamaterials

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Abstract

We present a method for homogenizing tensor transmission-line (TL) metamaterials. These are metamaterials consisting of loaded transmission-line networks, that can possess magnetically anisotropic (tensor) effective material parameters. The homogenization employs a local field averaging procedure to compute the effective material parameters. These effective material parameters can be dispersive or non-dispersive. For the tensor metamaterials possessing dispersive effective material parameters, the homogenization method takes advantage of the circuit topology of tensor transmission-line metamaterials to predict material parameters over a frequency range.

Keywords: Transmission-line metamaterial; Homogenization method; Anisotropic medium; Tensor material parameters

1. Introduction

Tensor transmission-line (TL) metamaterials have been proposed recently [1], and shown to exhibit the properties of magnetically anisotropic media [2]. The equivalence between tensor TL metamaterials and magnetically anisotropic media was established by comparing the dispersion equation of the metamaterial with that of a homogenous anisotropic medium. Here, we verify the magnetic anisotropy of tensor metamaterials by retrieving their effective material parameters through a homogenization procedure.

To date, various methods have been used to extract the effective material parameters of metamaterials. Analytical approaches that take into account frequency dispersion [3], mutual coupling effects [4] and spatial dispersion [5] have been proposed. Retrieval of effective material parameters from scattering measurements [6], and improved methods using the Bloch parameters of a metamaterial [7–9], and transition layers [8,10] have also been reported. In addition, numerical homogenization methods employing field averaging of eigenmode simulations [11] and others based on source driven problems [12,13] have been put forth. These methods have been applied to well known volumetric metamaterials ranging from microwave to optical frequencies.

Analytical homogenization methods for 1D [14] and 2D transmission-line metamaterials [15,16] as well as related 2D mushroom structures [17] have been studied in the past. Periodic analysis of these transmission-line metamaterials was performed in [15] and Bloch parameters of the same form as those in [8,9] were derived. However, field-averaging homogenization methods have not been applied to fullwave simulation data of transmission-line metamaterials.

In this paper, we present a method for calculating the effective medium parameters of tensor transmission-line metamaterials. The homogenization method averages the local electromagnetic field over the faces of the metamaterial's unit cell, in order to compute its effective material parameters. In addition, circuit representations of tensor TL metamaterial unit cells are used

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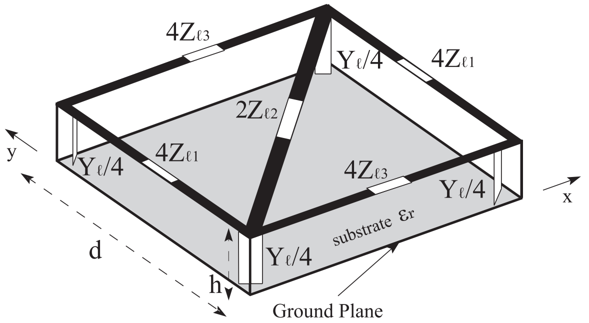

Fig. 1. Microstrip implementation of a tensor TL metamaterial unit cell with series and shunt loading elements.

to predict the frequency dispersion of their effective material parameters. The described homogenization technique is also applicable to isotropic and anisotropic transmission-line metamaterials that possess diagonal material parameters in the Cartesian basis [15,18–22]. The extracted material parameters are verified through full wave analysis.

2. Homogenization

Fig. 1 shows a microstrip implementation of a tensor TL metamaterial unit cell. The proposed homogenization procedure will be applied to this metamaterial. Fig. 2 depicts the set up that will be used to perform eigenvalue (periodic) simulations of the tensor TL metamaterial shown in Fig. 1, from which material parameters will be extracted. The set up includes periodic boundaries on the sidewalls of the unit cell and a perfectly matched layer on top to emulate unbounded free space. Integration paths shown on the faces of the unit cell will be used in the homogenization procedure. The eigenvalue problem is solved numerically using a commercial fullwave electromagnetic solver, Ansoft HFSS. The local E and H field distributions within the unit cell are computed numerically for a given set of phase delays (periodic conditions) in the x and y directions. By integrating the local fields on the faces of the unit cell, the following voltages V_x, V_y and currents I_x, I_y can be defined

$$V_x = -\int_{\ell 1} \bar{E} \cdot \bar{d}_\ell \quad V_y = -\int_{\ell 2} \bar{E} \cdot \bar{d}_\ell$$
$$I_x = \oint_{C_1} \bar{H} \cdot \bar{d}_\ell \qquad I_y = \oint_{C_2} \bar{H} \cdot \bar{d}_\ell. \tag{1}$$

where paths C_1 and C_2 represent the closed contours consisting of the lines 1–2–3–4 and 6–7–8–9, respectively. Although the integration paths C_1 and C_2 extend to the perfectly matched layer (PML) in the z direction (see Fig. 2), this is not necessary. One may simply ensure that the closed path integrals enclose the net current flowing along the microstrip lines in the respective directions. The lines along which the E field is integrated, ℓ_1 and ℓ_2, are shown as line 5 and line 10 in Fig. 2, respectively. Using the voltage and current expressions of (1), impedances Z_x, Z_y in x and y directions can be defined

$$Z_x = \frac{V_x}{I_x} \qquad Z_y = \frac{V_y}{I_y}. \tag{2}$$

The circuit parameters defined by (1) and (2) have a one-to-one correspondence to the field quantities of a *TEM* wave propagating in the parallel-plate waveguide of Fig. 3. The dielectric in the parallel-plate waveguide is assumed to have a permeability tensor $\bar{\bar{\mu}}$ and a scalar permittivity ϵ

$$\bar{\bar{\mu}} = \begin{pmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{pmatrix} \qquad \epsilon = \epsilon_z. \tag{3}$$

For the polarization of interest, the non-zero field quantities within the parallel-plate region are the z component of electric field E_z and the x and y components of magnetic field: H_x and H_y. Using these field components,

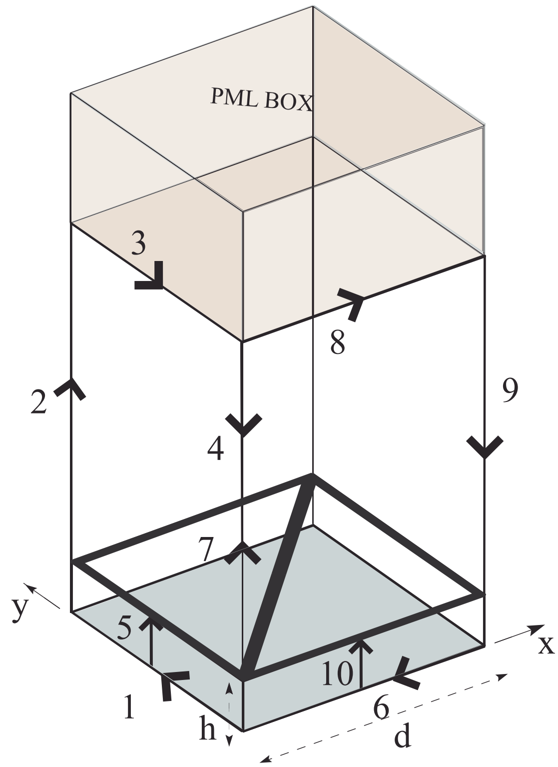

Fig. 2. The eigenmode simulation setup. The integration paths used in the homogenization procedure are shown.