

Nonlocal homogenization of an array of cubic particles made of resonant rings

M.G. Silveirinha^{a,*}, J.D. Baena^b, L. Jelinek^c, R. Marqués^c

^a Department of Electrical Engineering – Instituto de Telecomunicações, University of Coimbra, 3030 Coimbra, Portugal

^b Departamento de Física, Universidad Nacional de Colombia, Carrera 30 No 45-30 Bogotá D.C., Colombia

^c Departamento de Electrónica y Electromagnetismo, Universidad de Sevilla, 41012 Sevilla, Spain

Received 19 May 2009; received in revised form 24 August 2009; accepted 28 August 2009

Available online 8 September 2009

Abstract

Here, we develop a nonlocal homogenization model to characterize the electrodynamics of an array of cubic particles made of resonant rings. The effective parameters are calculated from the microscopic fields produced by a periodic external excitation. It is confirmed that the spatial dispersion effects cannot be neglected in the regime where $\mu \approx 0$. We demonstrate that when the array of resonant rings is combined with a triple wire medium formed by connected wires, the structure may behave approximately as an isotropic left-handed material.

© 2009 Elsevier B.V. All rights reserved.

PACS: 42.70.Qs; 78.66.Sq

Keywords: Homogenization; Spatial dispersion; Artificial magnetism

1. Introduction

Structured materials with a strong magnetic response have been under intense research in recent years [1–10], mainly due to their potential applications in the design of imaging systems with improved resolution [2–5]. In particular, it was recently demonstrated that a metamaterial lens formed by split-ring resonators (SRRs) boosts the sensitivity of the coil used in magnetic resonance imaging when operated in the regime $\mu = -1$ [6].

In most of the studies published in the literature, it is typically assumed from the outset that the response of the metamaterial is local, and, based on that assumption the effective parameters are usually calculated using the retrieval procedure reported in Ref. [11] (inversion of the scattering parameter data). Recently, in Ref. [12] the Lorentz local field theory was used to homogenize a metamaterial formed by an array of cubic particles with tetrahedral symmetry formed by split-ring resonators (a topology similar to that considered here), and the nonlocal magnetic permeability was calculated. It was demonstrated that the spatially dispersive model provides a unified description of the transverse electromagnetic waves and of the so-called magnetoinductive waves [13], demonstrating in this manner that the latter are a short-wavelength continuation of the former. In this work, we investigate a problem closely related to

* Corresponding author. Tel.: +351 239 796268;
fax: +351 239 796293.

E-mail addresses: mario.silveirinha@co.it.pt (M.G. Silveirinha),
jdbaad@unal.edu.co (J.D. Baena), l.jelinek@us.es (L. Jelinek),
marques@us.es (R. Marqués).

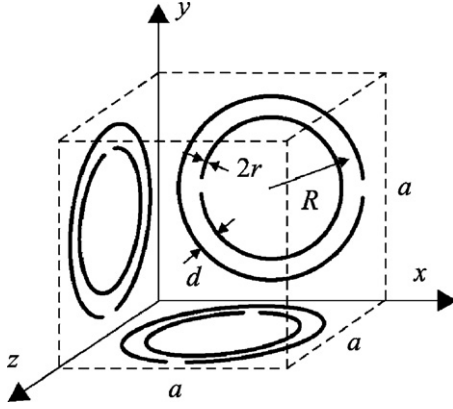


Fig. 1. Geometry of the unit cell of a structured material formed by resonant rings. The figure depicts the particular case in which the resonant particles are SRRs.

that considered in Ref. [12] but from a different perspective. Instead of applying the Lorentz's local field theory, we compute the nonlocal parameters using the homogenization method proposed in the works [14,15], which is based on the idea of exciting the metamaterial with a periodic source with suitable phase-shift. We obtain the exact solution (taking into account the interaction between all the particles in lattice) of the homogenization problem under the approximation that the response of the inclusions can be described using the dipole approximation, and an explicit formula for the nonlocal magnetic permeability is derived. The results of the analytical model are compared with the effective parameters obtained with full wave simulations that take into account all the details of the microstructure of the material. Finally, we study the electrodynamics of a system formed by the array of SRRs and a connected array of wires. It should be mentioned that other previous works (e.g. Ref. [16]) have studied the homogenization of arrays of SRRs taking into account rigorously the mutual effects and lattice ordering. The main contribution of our analysis, which extends our previous work [12], is the characterization of the spatial dispersion effects.

2. Homogenization

A representative geometry of the unit cell of the material under analysis is shown in Fig. 1. The unit cell contains three different resonant metallic rings, being each ring normal to one of the Cartesian axes. The lattice is simple cubic with lattice constant a . Clearly, each basic inclusion has an anisotropic response, but in the long wavelength limit the response of the composite material is approximately isotropic due to the spatial arrangement and orientation of the particles. The i th resonant ring in

the unit cell is by definition normal to the unit vector $\hat{\mathbf{u}}_i$ ($i = x, y, z$), and is centered at the point $\mathbf{r}_{0,i} = -(a/2) \hat{\mathbf{u}}_i$.

The resonant rings are generic planar (or quasi-planar) inclusions, which may be produced by some lumped or distributed capacitance. We suppose that the rings can be characterized by an impedance $Z_0 = j\omega L + 1/(j\omega C)$ obtained from a circuit model (for simplicity the effect of metallic loss is neglected; see Refs. [17,18] for the particular case of SRRs). The resonant rings will be modeled as dipole-type magnetic particles characterized by a uniaxial magnetic polarizability dyadic (tensor). The magnetic polarizability dyadic of the i th ring is

$$\bar{\bar{\alpha}}_i = \alpha_m \hat{\mathbf{u}}_i \hat{\mathbf{u}}_i, \quad (1)$$

where $\hat{\mathbf{u}}_i \hat{\mathbf{u}}_i \equiv \hat{\mathbf{u}}_i \otimes \hat{\mathbf{u}}_i$ represents the dyadic (tensor) product of two vectors. The parameter α_m (with unities [m^3]) is related to the impedance Z_0 as follows (including the effect of the radiation loss)

$$\begin{aligned} \alpha_m^{-1} &= \frac{Z_0}{-j\omega A^2 \mu_0} + j \frac{1}{6\pi} \left(\frac{\omega}{c} \right)^3 \\ &= \alpha_0^{-1} \left[\left(\frac{\omega_r}{\omega} \right)^2 - 1 \right] + j \frac{1}{6\pi} \left(\frac{\omega}{c} \right)^3, \end{aligned} \quad (2)$$

where $\alpha_0^{-1} = L/\mu_0 A^2$, A is the area of the ring and $\omega_r = 1/\sqrt{LC}$.

For simplicity, it will be assumed that the rings do not have an electric response (i.e. that the electric polarizability vanishes; in Section 3, we will discuss how to incorporate the electric response in the model). Thus, at a microscopic level, the magnetic dipole moment of the i th particle in the unit cell, $\mathbf{p}_i$, must verify:

$$\frac{\mathbf{p}_i}{\mu_0} = \bar{\bar{\alpha}}_i \cdot \mathbf{H}_{\text{loc}}(\mathbf{r}_{0,i}), \quad i = x, y, z, \quad (3)$$

where $\mathbf{H}_{\text{loc}}$ is the local magnetic field that polarizes the pertinent ring. The magnetic dipole $\mathbf{p}_i$ as defined above is related to the more traditional definition given in textbooks (e.g. Ref. [19]) as $\mathbf{m} = \mathbf{p}_i/\mu_0$.

2.1. Two models for the response of the rings

It is possible to model the response of the resonant rings using two alternative approaches. The first model takes into account that the rings are metallic particles, and thus that an external field induces a microscopic *electric current density* in each ring. Since it is assumed that the rings only have a magnetic response, which is necessarily caused by the vortex part of the induced electric current (artificial magnetism), the induced current is related to the magnetic dipole moments as follows (see

Download English Version:

<https://daneshyari.com/en/article/1532864>

Download Persian Version:

<https://daneshyari.com/article/1532864>

[Daneshyari.com](https://daneshyari.com)