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On the estimate of mixing length in interdigital micromixers

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Abstract

The multilamination process that characterizes interdigital micromixers is an efficient and technologically feasible method for maximizing and controlling mass and/or heat transfer between two or more segregated fluid streams. We analyze the dynamics of mixing that takes place in the mixing channel downstream the interdigital apparatus. Specifically, we investigate, for different flow profiles, how the channel length necessary to achieve a prescribed level of mixedness depends on the degree of lamination (number and thickness of lamellae) of the feed stream. As a case study, we consider plug, shear and Poiseuille flow, and compare steady-state profiles resulting from the numerical simulation of the full advection–diffusion problem with the analytical solution stemming from the one-dimensional Sturm–Liouville eigenvalue problem along the spanwise coordinate, obtained neglecting streamwise diffusion. We find that (i) the mixing length can be significantly affected by the flow profile, especially at high degree of lamination of the feed stream, and (ii) in general, no obvious scaling between mixing length and lamellar thickness can be assumed. A rigorous way to approach the design of these micromixers is proposed.

Keywords: Fluid mixing; Interdigital micromixers; Laminar flows; Homogenization; Lamination

1. Background and motivation

Interdigital micromixers provide a novel technology that allows to accomplish a nearly complete homogenization of two or more segregated fluid streams within extremely short contact times (e.g. up to order of milliseconds in the case of liquids, see [1] and therein cited references). Other uses of these devices include microreactions as microextraction [2], as well as liquid–liquid dispersion [3]. The simplest case of "T-junction" channel is also used for sensing and separating analytes [4,5], measuring diffusivities and determining kinetic rate constants [6].

The core of the equipment is constituted by a comb-like arrangement of microchannels that split the streams entering the system and rearrange them into a multilaminated structure where alternating *lamellae* of the two fluids are forced to flow alongside. The typical width of one of such lamellae is of the order of tens of micron. The spatially periodic structure of the process stream is then squeezed into a smaller channel, referred to as mixing channel, where the desired degree of mixing is reached.

Typically, the micromixers are identified by the type of geometry connecting the multilamination apparatus and the mixing channel. Frequently used configurations are the rectangular, slit-shaped, triangular, and the super-focus mixer [7,8]. Fig. 1 shows a schematic representation of the rectangular micromixer.

Apart from specific cases, such as that of relatively high Reynolds number flow (e.g. order $Re = 10^3$) in the slit-shaped micromixer, little mixing occurs in the portion of the apparatus connecting the multilamination device and the mixing channel. As a result, the flow stream at the entrance of the mixing channel can be thought of as an ordered array of alternating lamellae [8], each characterized by a constant concentration of one of the species that are to be mixed.

Beside lowering substantially the mixing time associated with advecting–diffusing species, the number and width of the lamellae can also have a strong influence on the performance of the device in the case where the two species are chemically reactive. As an example, numerical simulations suggest that the degree of lamination of reactants fed to a microreaction channel can impact substantially on the yield and product distribution of parallel-competitive and parallel-consecutive reactions [9].

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Nomenclature

Ai(z)	Airy function of first kind
Bi(z)	Airy function of second kind
C_h	generalized Fourier coefficients defined by Eq. (15)
$\mathcal{D}$	diffusivity
L	channel length
n_s	number of lamellae of the inlet concentration profile
Pe	Peclet number, $Pe = \delta^2 U/\mathcal{D}L$
s	lamellar width
U	characteristic mean velocity
$v_x(y)$	dimensionless axial velocity
x	dimensionless axial coordinate
y	dimensionless vertical coordinate
Y_h	eigenfunctions of the Sturm–Liouville problem

Greek letters

α	reciprocal of the aspect ratio, $\alpha = \delta/L$
δ	channel width
λ_h	eigenvalue of the Sturm–Liouville operator
μ	degree of mixedness
ξ	Pe-scaled axial coordinate $\xi = x/Pe$
$\sigma(x)$	variance of the concentration profile at x
σ_v	variance ratio defined by Eq. (6)
ϕ	dimensionless solute concentration
$\phi_i(y)$	inlet concentration profile
$\phi_i^{(n_s)}$	inlet lamellar profile defined by Eq. (4)

When designing micromixers, an important piece of information is an *a priori* estimate of the mixing performance associated with a given lamellar thickness, say s, of the feed stream. Specifically, one would like to estimate the length, L, of the channel that ensures a prescribed level of mixedness at the system outlet section.

An analytical estimate of the mixing length is usually obtained by invoking simplifying assumptions in the solution of the steady-state advection–diffusion problem, namely: (i) flat velocity profile (plug flow), (ii) high value of the channel aspect ratio (semi-infinite channel), and (iii) substitution of the zero diffusive flux condition at the channel walls with periodic boundary conditions in the spanwise direction[1].

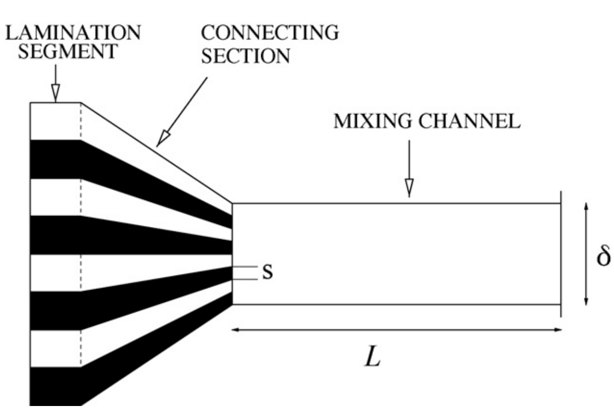

Fig. 1. Schematic diagram of the rectangular interdigital micromixer.

Under these assumptions, the advection–diffusion boundary value problem is recast into a simpler partial differential problem that is formally identical to a standard one-dimensional unsteady diffusion problem, where the relationship between time, say T, and length of the channel is given by the ratio between L and the convective mean velocity, say U, i.e. $T = L/U$. By using these approximations, a scaling relationship $T \sim s^2$ or, equivalently, $L \sim s^2$, is obtained (see, e.g. [2,11]). Furthermore, this scaling relationship results *independent of the level of mixedness* required (a derivation of this result is discussed in detail in Appendix A).

In this article, we challenge this estimate and analyze separately the role of each simplifying assumption. The analysis is carried out by using three prototypical flow profiles, namely plug, shear, and Poiseuille flow, which are frequently encountered in microhydrodynamic applications. Specifically, plug-like velocity profiles are encountered in electroosmotically driven flows [12], while the shear profile, typical of micromotors and microbearings [13], can in principle be obtained by electroosmosis through an asymmetric surface treatment of the channel walls. The Poiseuille profile characterizes pressure-driven microflows.

We compare and contrast results from direct numerical simulations of the full advection–diffusion equation with the solution of the simplified transport equation stemming from the high-aspect ratio approximation, which is equivalent to a one-dimensional unsteady diffusion problem with variable diffusivity along the spanwise coordinate. The analysis of this problem reduces to finding the eigenvalue–eigenfunction spectrum of a Sturm–Liouville second-order problem specified by the flow profile.

The article is organized as follows. Section 2 introduces the physical problem, and its mathematical and computational formulation. Section 3 addresses the Sturm–Liouville approach for high-aspect ratio microchannels, and the spectral (eigenvalue/eigenfunction) structure of the resulting advection–diffusion operator. Specifically, Section 3.3 derives the main results for typical laminar flows in microchannels, and Section 3.4 introduces a quantitative way for the design of interdigital micromixers based on the Sturm–Liouville formulation discussed in Section 3.3. It is shown that the classical scaling law $L \sim s^2$ (relating the channel length to the lamellar width of the feed stream) it is far from being verified for generic laminar flows. The approach proposed in Section 3.4 provides a simple design strategy for interdigital micromixers. In Appendix

[1] The simplifying assumptions of flat velocity profile and semi-infinite channel (which allow to neglect axial diffusion) are also used to construct one-dimensional models of mixing for predicting the mixing length associated with more complex geometries, such as, e.g. the split-and-recombine mixer [10]. In this context, the effort directed towards the construction of simplified analytical models of the (possibly chaotic) three-dimensional mixing process is motivated by the unavoidable presence of spurious diffusion, which can overshadow the results of numerical simulations when complex flows are dealt with.