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Extracting the bulk effective parameters of a metamaterial via the scattering from a single planar array of particles

Aaron D. Scher, Edward F. Kuester *

Department of Electrical and Computer Engineering, University of Colorado at Boulder, Boulder, CO 80309, USA

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Abstract

In this paper we present a method for retrieving the effective parameters of a metamaterial composed of a regular rectangular orthorhombic lattice of linear biaxially anisotropic particles suspended in free space. By assuming the point–dipole interaction approximation, equations are derived which extract the electric and magnetic polarizabilities of the individual particles given the measured or simulated scattering parameters of a single planar array of particles. These results are in turn substituted into the Clausius–Mossotti equations to find the bulk effective permittivity and effective permeability. To demonstrate our approach, the extraction method is applied to a metamaterial consisting of a cubic arrangement of magnetodielectric spheres using the scattering parameters obtained by simulating the structure with Ansoft HFSS. Our results show good agreement with a known analytical solution at frequencies in which the Clausius–Mossotti approximation is valid.

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1. Introduction and objectives

In recent years there has been much interest in composite electromagnetic structures known as metamaterials. Due to finer scale interactions, such structures can be designed to exhibit bulk or macroscopic properties which are fundamentally different than those of the underlying conventional materials in the composition. While there are many ways of constructing a metamaterial, one basic design, which in a sense imitates the structure of naturally occurring crystals, consists of a periodic arrangement of polarizable “particles” such as split ring resonators [1] or magnetodielectric spheres [2] suspended in a background host-medium. A common and useful practice when designing or analyzing such structures is to view the structure as a whole as an effective medium with an effective relative electric permittivity $\varepsilon_{r,\mathrm{eff}}$ and relative magnetic permeability $\mu_{r,\mathrm{eff}}$ (or equivalently, with an effective wave impedance $\eta_{\mathrm{eff}} = \eta_0(\mu_{r,\mathrm{eff}}/\varepsilon_{r,\mathrm{eff}})^{1/2}$ and effective index of refraction $n_{\mathrm{eff}} = (\mu_{r,\mathrm{eff}}\varepsilon_{r,\mathrm{eff}})^{1/2}$). This approach is justified by homogenization and generally holds true if the spatial dimensions of the basic unit cell are sufficiently small compared to both the wavelength of the fields supported by the structure and that of free space at the operating frequency. Interestingly, if the operating frequency is near the resonant frequency of the individual constituent particles, then certain structures can be designed to behave as a double negative medium, meaning that the real parts of both μ_{eff} and $\varepsilon_{\mathrm{eff}}$ are simultaneously negative. This

* Corresponding author. Tel.: +1 303 492 5173;
fax: +1 303 492 2758.
E-mail address: kuester@schof.colorado.edu (E.F. Kuester).

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property is usually not found in nature and has novel and exciting applications in lens, antenna, and device design.

In order to verify theoretical predictions, it is important to be able to measure these effective electromagnetic properties. The Nicolson–Ross method [3] is a well-established method for performing such measurements on ordinary materials. It uses measured values of scattering parameters (reflection and transmission coefficients) from a slab of material under test, and from the equations describing this scattering extracts the propagation constant and wave impedance. The equations are based on the Fresnel reflection and transmission coefficients at a material interface, which in turn are obtained from the assumption that the tangential electric and magnetic fields **E** and **H** are continuous at the interface.

The Nicolson–Ross method has been adapted for measurement of metamaterials by Smith et al. [4,5], using the measured or simulated scattering parameters for a finite number of cascaded plane layers of particles making up the composite. The scattering parameters are related to the effective medium parameters by assuming that the structure can be replaced by an effective medium slab with an equivalent thickness. Now, it is known that the assumption of continuity of the macroscopic tangential **E** and **H** fields is only an approximation, because near an interface, fields within a crystal behave differently than they do deep within the bulk medium [6]. As the thickness is increased, these boundary effects become less important, and the results tend to converge to the bulk medium properties. It has therefore been recommended to include at least three or four unit cells in the analysis. However, the method has an inherent limitation that allows it to predict the real part of the propagation constant only to within an additive integer multiple of π/l, where l is the thickness of the sample. Hence, in order to reduce the ambiguity of the integer, it is also recommended that l be small; i.e., that there should not be too many unit cells included in the analysis. The nature of this ambiguity is only exacerbated by those metamaterials which are based on resonant elements, because near resonance the magnitude of the effective refractive index can become quite large, which can result in an effective electrical length for a single unit cell comparable to a wavelength. In fact, one must question the very validity of the effective medium assumption at such frequencies.

As highlighted by Simovski and Tretyakov [7,8], a fundamental problem with the Nicolson–Ross type of approach is that it uses a local model (meaning that the effective parameters are assumed to hold true up to the air–metamaterial interface and are independent of the wavevector), whereas the response of a real composite is inherently nonlocal. Using the point–dipole interaction model, in Refs. [7,8] the meaning and common misconceptions behind the notions of "local" and "nonlocal" material parameters are clarified, and a retrieval algorithm is proposed which relates the measured scattered data of a test slab to the Bloch impedance and wavenumber of the periodic structure. The Bloch impedance and wavenumber are in turn related to the local material parameters by a simple homogenization scheme.

In this paper, we propose a more direct method for extracting the bulk effective parameters for the case in which the metamaterial is composed of a rectangular lattice formed by linear biaxially anisotropic particles. By construction, the proposed method requires only the scattering parameters from a single planar array of particles for an incident field polarized along each principal axis. Instead of looking at the array as an equivalent slab with an effective thickness equal to that of a single unit cell (as in the Nicolson–Ross method), we will consider the system in the point dipole approximation (as done in Refs. [7,8]) and solve directly for the components of the diagonal electric-dipole and magnetic-dipole polarizability tensors. The effective medium parameters are then found by substituting the extracted polarizability values into the Clausius–Mossotti equations. As discussed in more detail later, the polarizabilities extracted using our method can also be incorporated into a more sophisticated characterization which takes into account spatial dispersion.

2. The effective medium approach

We begin by considering a metamaterial structure composed of linear, biaxially anisotropic, nonchiral scatterers or particles arranged in a three-dimensional rectangular lattice. The background medium is air or free space. The periods along the x-, y-, and z-axes are equal to a, b, and d, respectively. To simplify the problem we use the point–dipole interaction approximation; meaning that the fields produced by an individual particle are assumed to be equal to that produced by a point–dipole located at the center of the particle. This approximation requires that contributions from higher order multipoles be negligible. Hence, we restrict our analysis to particles with dimensions sufficiently smaller than a wavelength of the field inside the medium so that the variation of local fields over the volume of the particle can be ignored. We also assume that the sizes of the particles are sufficiently smaller than the lattice period to ensure that dipolar interaction is valid. For spherical particles, the theory is applicable even as the diameter of the spheres approaches the lattice constant. However, for other types