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# Scattering of electromagnetic plane wave from a perfect electric conducting strip placed at interface of topological insulator–chiral medium

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ABSTRACT

In this manuscript, scattering from a perfect electric conducting strip located at planar interface of topological insulator (TI)–chiral medium is investigated using the Kobayashi Potential method. Longitudinal components of electric and magnetic vector potential in terms of unknown weighting function are considered. Use of related set of boundary conditions yields two algebraic equations and four dual integral equations (DIEs). Integrand of two DIEs are expanded in terms of the characteristic functions with expansion coefficients which must satisfy, simultaneously, the discontinuous property of the Weber–Schafheitlin integrals, required edge and boundary conditions. The resulting expressions are then combined with algebraic equations to express the weighting function in terms of expansion coefficients, these expansion coefficients are then substituted in remaining DIEs. The projection is applied using the Jacobi polynomials. This treatment yields matrix equation for expansion coefficients which is solved numerically. These unknown expansion coefficients are used to find the scattered field. The far zone scattering width is investigated with respect to different parameters of the geometry, i.e, chirality of chiral medium, angle of incidence, size of the strip. Significant effects of different parameters including TI parameter on the scattering width are noted.



## 1. Introduction

Reflection, refraction and scattering phenomena are very common and important in nature, so cannot be ignored while studying electromagnetics. Electromagnetic waves experience reflection and refraction phenomena when planar interface due to different media is encountered by these waves [1,2]. Similarly, scattering is required to be discussed for rough, curved and surfaces having edges [3–6]. There are different numerical and analytical high frequency techniques which are widely discussed in the literature. For instance, Geometric Theory of Diffractions, Physical Optics and Physical Theory of Diffraction are few high frequency asymptotic methods [7,8]. On the other hand, Method of Moment and Finite Difference Time Domain method are numerical techniques.

Kobayashi Potential (KP) method is a semi-analytical technique which can be used to deal with the mixed boundary value problems. This method was introduced by Iwao Kobayashi in 1930 and was titled as Kobayashi potential by I.N. Sneddon [9]. Iwoa Kobayashi dealt with potential problems of different kinds like circular, semi-circular disks and electrified conducting strip. In this method, edge conditions and boundary conditions are fulfilled by using discontinuous property of Weber–Schafheitlin Integral (WSI) [10–13]. Jacobi polynomials are also used in KP method to satisfy some edge conditions.

Scattering of electromagnetic waves rely not only on the scatterer size but also on material properties of host medium where scatterer is to be placed. Due to this attractive feature, scatterers made of and deposited in artificial materials have grasped significant interest of many researchers [14–18]. Topological insulator (TI) medium has snatched interest of different researchers due to their exotic properties. For instant, TI corresponds to ordinary insulators in the bulk whereas their surface states are metallic and sheltered topologically by time-reversal symmetry. In many material systems, TI have been predicted theoretically and experimental evidence exists [19,20]. In three dimensional TI, electromagnetic wave behavior can be interpreted by using Lagrangian containing conventional Maxwell term and extra term due to Transverse Magnato Electric (TME) effects, $t_i = \frac{\theta\alpha}{\pi}$, where $\alpha$ is the fine structure constant and $\theta$ is the quantized angular variable

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used to characterized the TME effects [21]. For the finite value of $\theta$ time reversal symmetry is broken, which can be controlled by covering TI with a thin magnetic layer [22]. TME response can be incorporated in constitutive relations for TI medium like chiral medium. Constitutive relations for TI medium can be described as follows [16,21],

$$\mathbf{D} = \epsilon_1 \mathbf{E} + t_i \mathbf{B} \tag{1a}$$

$$\mathbf{H} = \frac{1}{\mu_1}\mathbf{B} - t_i \mathbf{E} \tag{1b}$$

where $\epsilon_1$ and $\mu_1$ are permittivity and permeability of TI medium, respectively. It may be noted that CGS system of units has been used in this paper. Both of these are dimensionless quantities in CGS system of units. In general, any linear polarized plane wave can be decomposed into two circularly polarized plane waves [23], so electric field and magnetic flux density inside the TI medium can be written as

$$\mathbf{E} = \mathbf{E}_R + \mathbf{E}_L \tag{2a}$$

$$\mathbf{B} = \mathbf{B}_R + \mathbf{B}_L \tag{2b}$$

Each set, $\{(\mathbf{E}_R, \mathbf{B}_R), (\mathbf{E}_L, \mathbf{B}_L)\}$, must satisfy the Maxwell equations independently, with perception that no coupling exists between the fields for two waves [23], that is

$$\nabla \times \mathbf{E}_{\substack{R\\L}} = \frac{i\omega}{c}\mathbf{B}_{\substack{R\\L}} \tag{3}$$

Using above Maxwell equations, the magnetic flux density can be determined and is written as follows,

$$\mathbf{B} = -i\sqrt{\mu_1\epsilon_1}[\mathbf{E}_L - \mathbf{E}_R] \tag{4}$$

The corresponding magnetic field can be determined from (1b) and is written below,

$$\mathbf{H} = \left(\frac{i}{\eta_1} - t_i\right)\mathbf{E}_R - \left(\frac{i}{\eta_1} + t_i\right)\mathbf{E}_L \tag{5}$$

It may be noted that relations given in (2), (4) and (5) had been used in published literature treating TI medium [21,24–27]. Using (4) in Maxwell equations yields the following relation,

$$\nabla \times \mathbf{E}_{\substack{R\\L}} = \mp \kappa \mathbf{E}_{\substack{R\\L}} \tag{6}$$

where $\kappa = \frac{\omega}{c}\sqrt{\mu_1\epsilon_1}$ is the wave number and $c$ the velocity of light in TI medium.

Chiral medium can artificially be constructed by inserting randomly oriented chiral objects in a host dielectric medium. In principle, the chiral object possesses mirror-asymmetry. Human hands are the most universal example of chiral objects. Chiral medium provides one more degree of freedom to control the EM waves phenomena, i.e., chiral parameter or chirality. Due to this extraordinary nature of chiral medium, this medium is the subject of interest of many researchers during the last few decades. Scattering and propagation behavior of electromagnetic wave from chiral medium is discussed [28–30]. In order to model the electromagnetic waves in chiral medium by using the Maxwell equations and constitute relations, an additional constitutive parameter or term in constitutive relations is used. In present discussion, Drude–Born–Fedorov (DBF) constitutive relations to describe chiral medium are used. DBF relations are written below [23,31]

$$\mathbf{D} = \epsilon \mathbf{E} + \epsilon\beta\nabla \times \mathbf{E} \tag{7a}$$

$$\mathbf{B} = \mu \mathbf{H} + \mu\beta\nabla \times \mathbf{H} \tag{7b}$$

where $\epsilon$ and $\mu$ are, respectively, permittivity and permeability of chiral medium both of which are dimensionless quantities in CGS system, $\beta$ is the chirality parameter for chiral medium. The electric and magnetic fields in-terms of Beltrami fields, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, may be expressed as

$$\mathbf{E} = \mathbf{Q}_1 - i\eta\mathbf{Q}_2 \tag{8a}$$

$$\mathbf{H} = \frac{i}{\eta}\mathbf{Q}_1 + \mathbf{Q}_2 \tag{8b}$$

where $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are left and right circularly polarized waves in chiral medium, respectively. In above relations, the term '$\eta = \sqrt{\mu/\epsilon}$' is intrinsic impedance of the chiral medium. Beltrami fields in chiral medium are divergence free quantities and their polarization can be described by the following circulation equations

$$\nabla \times \mathbf{Q}_{\substack{1\\2}} = \pm \gamma_{\substack{1\\2}} \mathbf{Q}_{\substack{1\\2}} \tag{9}$$

where $\gamma_1$ and $\gamma_2$ are the wavenumbers for left and right handed Beltrami fields, respectively. These can be defined as $\gamma_{\substack{1\\2}} = \frac{k}{1 \mp k\beta}$, where $k = \frac{\omega}{c}\sqrt{\mu\epsilon}$. The propagation of light beam in chiral nematic liquid crystal had been predicted theoretically and verified experimentally [32,33]. These have a lot of applications in optical signal processing and switching. It may also be known as optically active medium which is promising for many applications in the field of antennas, radar and waveguides.

In this manuscript, scattering from a perfectly conducting strip placed at TI-chiral medium has been investigated. Thick magnetic coating has been assumed to break the time reversal symmetry for TI medium [15,21]. The TI and chiral media have exotic applications in electromagnetics, so the motivation is to investigate the scattering from perfectly conducting strip at the interface of two prodigious media. Importance of grating and micro-strip antennas cannot be denied in electromagnetics. Many researchers had published a lot in this aspect [31,34–37]. For all the above mentioned geometries, scattering from strip geometry is a basic unit. Proposed geometry can be reduced to free space-chiral geometry by taking $\theta = 0$, for TI medium. After abridging TI medium into free space, our results agree well with published work [12]. The time

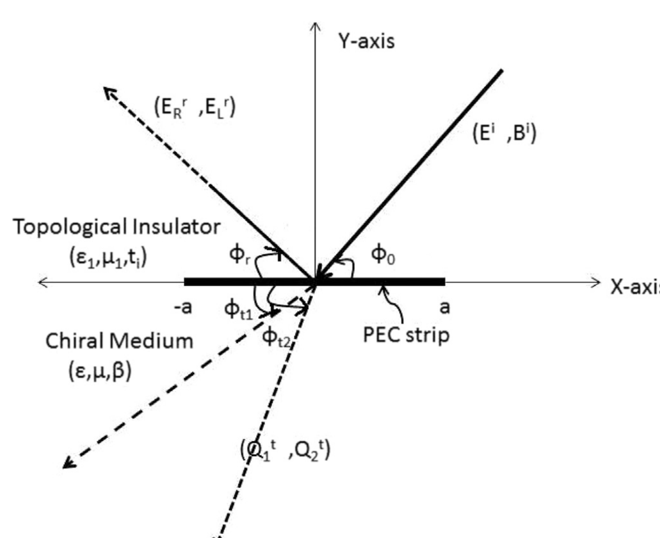


**Fig. 1.** A PEC strip placed at interface of TI–chiral medium.