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Shaped beam scattering by a spheroidal object

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A R T I C L E I N F O

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A theoretical procedure is developed for the calculation of the electromagnetic fields scattered by a spheroidal object with arbitrary monochromatic illumination. The suggested solution utilizes the method of moments technique in a spheroidal coordinate system. For oblique incidence of a Gaussian beam and zero-order Bessel beam, numerical results of the normalized differential scattering cross section are presented, and the scattering characteristics are analyzed concisely.

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1. Introduction

The problem of electromagnetic (EM) scattering by a spheroidal object, for incidence of a plane wave or a Gaussian beam, has been investigated extensively by so many researchers. Based on the fundamental method of separation of variables for the vector wave functions in the spheroidal coordinate system, Asano et al. studied in detail the EM scattering of a linearly polarized plane wave by a homogeneous isotropic spheroid with any size and refractive index [1,2]. The case of a multilayered spheroid has been treated by the extended boundary condition method (EBCM) [3]. For an incident Gaussian beam, Barton has calculated the intensity distributions internal and external to a spheroid with arbitrary illumination [4,5]. Within the generalized Lorenz–Mie theory (GLMT) framework, Han et al. provided an analytical solution to the scattering of an on-axis and off-axis Gaussian beam by a spheroidal object [6,7].

Recently, we have analyzed the scattering of a Gaussian beam for oblique incidence on a spheroidal object, by expanding the Gaussian beam as an infinite series of spheroidal vector wave functions (SVWFs) [8,9]. In our method, it is necessary to have the SVWFs expansion of the incident Gaussian beam. But, such an expansion is usually too difficult to obtain for some shaped beams as the zero-order Bessel beam (ZOBB), Hertzian electric dipole radiation, and so on. In this paper, based on the method of moments (MoM) procedure, an exact semi-analytical solution is presented to the scattering of an arbitrarily shaped beam by a spheroidal object.

The paper is organized as follows. In Section 2, a theoretical procedure is provided for the determination of the scattered fields of a shaped beam by a spheroidal object. Section 3 discusses concisely the scattering characteristics of a Gaussian beam and a ZOBB. The main findings are summarized in Section 4.

2. Formulation

As schematically shown in Fig. 1, an arbitrarily shaped beam propagates in free space and from the negative z' to the positive z' axis in its own Cartesian coordinate system $O'x'y'z'$. The system $Ox''y''z''$ is parallel to $O'x'y'z'$, with origin O having the Cartesian coordinates (x_0, y_0, z_0) in $O'x'y'z'$. A spheroidal object (semifocal distance, semimajor and semiminor axes respectively denoted by f, a and b) is natural to the system $Oxyz$, which is obtained by rotating $Ox''y''z''$ through a single Euler angle β [10]. In this paper, a time dependence of the form $\exp(-i\omega t)$ is assumed and suppressed for the EM fields.

According to the radiation condition of an outgoing wave, an appropriate expansion of the scattered fields in terms of the SVWFs with respect to $Oxyz$ can be written as [9,11]

$$\mathbf{E}^s = E_0 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} i^n \Big[\alpha_{mn}\ \mathbf{M}_{emn}^{r(3)}(c) + \alpha'_{mn}\ \mathbf{M}_{omn}^{r(3)}(c) + i\beta'_{mn}\mathbf{N}_{emn}^{r(3)}(c) + i\beta_{mn}\mathbf{N}_{omn}^{r(3)}(c) \Big] \quad (1)$$

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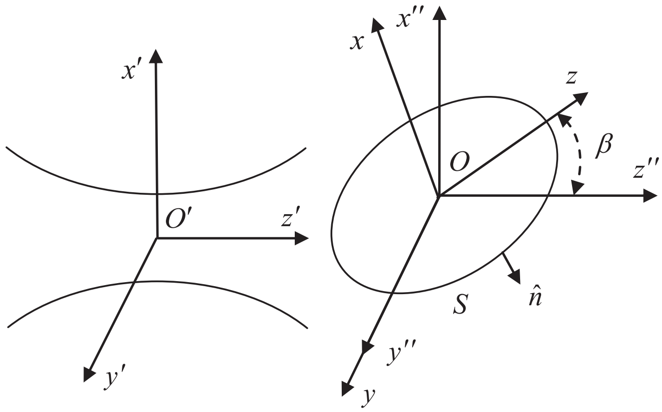

Fig. 1. A spheroidal object illuminated by an arbitrarily shaped beam.

$$\mathbf{H}^s = -i\frac{E_0}{\eta_0}\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\alpha_{mn}\mathbf{N}^{r(3)}_{emn}(c) + \alpha'_{mn}\mathbf{N}^{r(3)}_{omn}(c) + i\beta'_{mn}\mathbf{M}^{r(3)}_{emn}(c) + i\beta_{mn}\mathbf{M}^{r(3)}_{omn}(c)\Big] \tag{2}$$

where $c = kf$, and k, η_0 are the free space wave number and wave impedance.

For a dielectric spheroidal object, the internal fields can be expanded in terms of appropriate SVWFs as follows:

$$\mathbf{E}^w = E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\mathbf{M}^{r(1)}_{emn}(c_1) + \delta'_{mn}\mathbf{M}^{r(1)}_{omn}(c_1) + i\gamma'_{mn}\mathbf{N}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\mathbf{N}^{r(1)}_{omn}(c_1)\Big] \tag{3}$$

$$\mathbf{H}^w = -i\frac{E_0}{\eta_1}\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\mathbf{N}^{r(1)}_{emn}(c_1) + \delta'_{mn}\mathbf{N}^{r(1)}_{omn}(c_1) + i\gamma'_{mn}\mathbf{M}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\mathbf{M}^{r(1)}_{omn}(c_1)\Big] \tag{4}$$

where $c_1 = fk_1$, $k_1 = k\tilde{n}$, $\eta_1 = \eta_0/\tilde{n}$, and $\tilde{n}$ is the refractive index of the material of the dielectric spheroid relative to that of free space.

In the MoM scheme, Eqs. (1)–(4) can be interpreted that the expansions of the scattered and internal fields are obtained using appropriate SVWFs as basis functions.

The boundary conditions require that the tangential components of the EM fields be continuous

$$\hat{n}\times(\mathbf{E}^s + \mathbf{E}^i) = \hat{n}\times\mathbf{E}^w \tag{5}$$

$$\hat{n}\times(\mathbf{H}^s + \mathbf{H}^i) = \hat{n}\times\mathbf{H}^w \tag{6}$$

where $\mathbf{E}^i$ and $\mathbf{H}^i$ respectively represent the electric and magnetic fields of the incident shaped beam, and $\hat{n}$ denotes the outward normal to the spheroidal object's surface S.

Substituting Eqs. (1)–(4) into Eqs. (5) and (6), we can obtain

$$\hat{n}\times\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\alpha_{mn}\mathbf{M}^{r(3)}_{emn}(c) + \alpha'_{mn}\mathbf{M}^{r(3)}_{omn}(c) + i\beta'_{mn}\mathbf{N}^{r(3)}_{emn}(c) + i\beta_{mn}\mathbf{N}^{r(3)}_{omn}(c)\Big] + \hat{n}\times\mathbf{E}^i = \hat{n}\times\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\mathbf{M}^{r(1)}_{emn}(c_1) + \delta'_{mn}\mathbf{M}^{r(1)}_{omn}(c_1) + i\gamma'_{mn}\mathbf{N}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\mathbf{N}^{r(1)}_{omn}(c_1)\Big] \tag{7}$$

$$\hat{n}\times\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\alpha_{mn}\mathbf{N}^{r(3)}_{emn}(c) + \alpha'_{mn}\mathbf{N}^{r(3)}_{omn}(c) + i\beta'_{mn}\mathbf{M}^{r(3)}_{emn}(c) + i\beta_{mn}\mathbf{M}^{r(3)}_{omn}(c)\Big] + i\eta_0\hat{n}\times\mathbf{H}^i = \hat{n}\times\frac{\eta_0}{\eta_1}\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\mathbf{N}^{r(1)}_{emn}(c_1) + \delta'_{mn}\mathbf{N}^{r(1)}_{omn}(c_1) + i\gamma'_{mn}\mathbf{M}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\mathbf{M}^{r(1)}_{omn}(c_1)\Big] \tag{8}$$

Multiplication of Eq. (7) with $\mathbf{M}^{r(1)}_{om'n'}(c_1)$, $\mathbf{N}^{r(1)}_{em'n'}(c_1)$ respectively and integration over the surface S lead to

$$\sum_{n=m}^{\infty} i^n\Big[\alpha_{mn}\oint_S\mathbf{M}^{r(1)}_{om'n'}(c_1)\times\mathbf{M}^{r(3)}_{emn}(c) + i\beta_{mn}\oint_S\mathbf{M}^{r(1)}_{om'n'}(c_1)\times\mathbf{N}^{r(3)}_{omn}(c)\Big]\cdot\hat{n}dS + \oint_S\mathbf{M}^{r(1)}_{om'n'}(c_1)\times\mathbf{E}^i\cdot\hat{n}dS = \sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\oint_S\mathbf{M}^{r(1)}_{om'n'}(c_1)\times\mathbf{M}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\oint_S\mathbf{M}^{r(1)}_{om'n'}(c_1)\times\mathbf{N}^{r(1)}_{omn}(c_1)\Big]\cdot\hat{n}dS \tag{9}$$

$$\sum_{n=m}^{\infty} i^n\Big[\alpha_{mn}\oint_S\mathbf{N}^{r(1)}_{em'n'}(c_1)\times\mathbf{M}^{r(3)}_{emn}(c) + i\beta_{mn}\oint_S\mathbf{N}^{r(1)}_{em'n'}(c_1)\times\mathbf{N}^{r(3)}_{omn}(c)\Big]\cdot\hat{n}dS + \oint_S\mathbf{N}^{r(1)}_{em'n'}(c_1)\times\mathbf{E}^i\cdot\hat{n}dS = \sum_{n=m}^{\infty} i^n\Big[\delta_{mn}\oint_S\mathbf{N}^{r(1)}_{em'n'}(c_1)\times\mathbf{M}^{r(1)}_{emn}(c_1) + i\gamma_{mn}\oint_S\mathbf{N}^{r(1)}_{em'n'}(c_1)\times\mathbf{N}^{r(1)}_{omn}(c_1)\Big]\cdot\hat{n}dS \tag{10}$$

In deriving Eqs. (9) and (10), we have used the well-known orthogonality relations among the trigonometric functions $\sin m\phi$ and $\cos m\phi$, and considered the following expression

$$\hat{n}dS = f^2\left(\zeta^2 - 1\right)^{\frac{1}{2}}\left(\zeta^2 - \eta^2\right)^{\frac{1}{2}}\hat{\zeta}\,d\eta d\phi \tag{11}$$

where $\zeta = a/f$ (a constant over the surface of the spheroid S), $\eta \in [-1, 1]$ and $\varphi \in [0, 2\pi]$ are the radial, angular and azimuthal coordinates in the spheroidal coordinate system, and $\hat{\zeta}$, equivalent to $\hat{n}$, is the outward unit normal vector.

Multiplicating Eq. (8) with $\mathbf{M}^{r(1)}_{em'n'}(c_1)$, $\mathbf{N}^{r(1)}_{om'n'}(c_1)$ and integrating over the surface S, we can have

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