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Changes in the spectral degree of coherence of a light wave on scattering from a particulate medium

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ABSTRACT

The spectral degree of coherence of light wave on scattering from a collection of particles is discussed. It is shown that both the characteristic of each particle and the distribution of particles in the collection play roles in the spectral degree of coherence of the far-zone scattered field. Two special cases, i.e. a collection of random particles with determinate distribution and a collection of determinate particles with random distribution, are discussed, and the particle-induced coherence change and the distribution-induced coherence change are found in the scattered field.

1. Introduction

The weak scattering theory, which discusses the relationship between the properties of the scattered field and the characteristics of the scattering medium, is a subject of considerable importance due to its potential applications in areas such as remote sensing, detection, and medical diagnosis. During the past three decades, the scattering of light waves, both from a continuous medium and from a particulate medium, has been discussed [1–6]. It has been shown that the characteristics of the scattering medium is an important factor that may influence the properties of the scattered field (for a review of these studies, see [7]). Recently, the incident light wave of the scattering problems has been generalized from plane light wave to more commonly used light, for example, stochastic electromagnetic waves [8–10], partially coherent light [11], vortex beam [12], and plane wave pulse [13]. It has been shown that the property of the incident field is also an important factor that play roles in determining the properties of the scattered field.

In the discussion of light wave scattering, a specific type of scatterer, i.e. the collection of particles, have attracted much attention. For examples, the spectral change of a polychromatic light wave on scattering from a collection of particles has been discussed by Dogariu and Wolf [14], and the effect of the pair-structure factor of a collection of particles on the scattered field has been discussed by Sahin and Korotkova [15]. In addition, the possibility for the determination of the structure characteristics from the measurement of scattered field has also been discussed (see, for examples [16,17]). It has been shown that the spectral coherence of the far-zone scattered field is closely related with the characteristics of the collection. This phenomenon may provide a potential method to determine the structure feature of a scattering medium. However, to the best of our knowledge, almost all of the discussion is based on the assumption that the particles in the collection are determinate. In this manuscript, we extend the discussion to a more general form, i.e. a collection of random particles with random distribution. The changes of the spectral degree of coherence of light waves on scattering from a collection of particles will be discussed, and two special cases, i.e. a collection of random particles with determinate distribution and a collection of determinate particles with random distribution, will be presented.

2. Theory

As shown in Fig. 1, we consider a spatially coherent plane wave, with a propagation direction specified by a real unit vector $\mathbf{s}_0$, is incident on a scattering medium. The property of the incident field at a pair of points $\mathbf{r}'_1$ and $\mathbf{r}'_2$ within the area of the scatterer can be described by its cross-spectral density function with a form of [18]

$$W^{(i)}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = S^{(i)}(\omega)\exp\left[ik\mathbf{s}_0\cdot(\mathbf{r}'_2 - \mathbf{r}'_1)\right], \tag{1}$$

where $S^{(i)}(\omega)$ is the spectrum of the incident light wave, ω is the angular frequency of light, and $k = \omega/c$ with c being the speed of light in vacuum. The spectral degree of coherence of the incident

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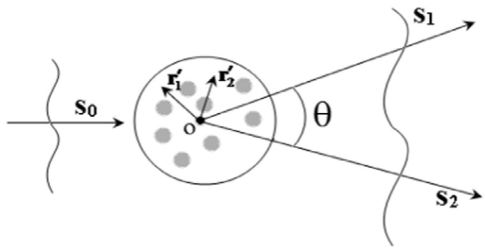

Fig. 1. Illustration of notations.

field can be obtained from its cross-spectral density function, by the following definition [18]

$$\mu^{(i)}(\mathbf{r}_1', \mathbf{r}_2', \mathbf{s}_0, \omega) = \frac{W^{(i)}(\mathbf{r}_1', \mathbf{r}_2', \mathbf{s}_0, \omega)}{\sqrt{S^{(i)}(\mathbf{r}_1', \mathbf{s}_0, \omega)}\sqrt{S^{(i)}(\mathbf{r}_2', \mathbf{s}_0, \omega)}}, \tag{2}$$

where $S^{(i)}(\mathbf{r}', \mathbf{s}_0, \omega)$ represents the spectral density of the incident field. On substituting from Eq. (1) into Eq. (2), one can obtain the spectral degree of coherence of the incident field with a form of

$$\mu^{(i)}(\mathbf{r}_1', \mathbf{r}_2', \mathbf{s}_0, \omega) = \exp\left[ik\mathbf{s}_0\cdot(\mathbf{r}_2' - \mathbf{r}_1')\right]. \tag{3}$$

As shown in Eq. (3), since the modulus of the spectral degree of coherence for any pair of points $\mathbf{r}_1'$ and $\mathbf{r}_2'$ are equal to unity, the incident light wave is spatially completely coherent at the frequency ω throughout the whole space.

Assume that the scatterer is not a continuous medium but is comprised of a collection of random distributed particles that are identical to each other. The correlation function of the scattering potentials of the entire collection is defined as [18]

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = \left\langle F^*(\mathbf{r}_1', \omega)F(\mathbf{r}_2', \omega)\right\rangle, \tag{4}$$

where $F(\mathbf{r}', \omega)$ is the scattering potential of the entire collection, with a form of

$$F(\mathbf{r}', \omega) = \sum_m f(\mathbf{r}' - \mathbf{r}_m', \omega), \tag{5}$$

where $f(\mathbf{r}', \omega)$ is the scattering potential of each particle with $\mathbf{r}_m'$ being the center location vector of the m-th particle. For the convenience of the following discussion, let us rewrite the scattering potential of the whole collection as

$$F(\mathbf{r}', \omega) = f(\mathbf{r}', \omega) \otimes \sum_m \delta(\mathbf{r}' - \mathbf{r}_m'), \tag{6}$$

where "$\otimes$" denotes the convolution operation, and $\delta(\cdots)$ denotes the Dirac delta function. On substituting from Eq. (6) into Eq. (4), and after some re-arrangement, one can find

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = \left\langle \left[f^*(\mathbf{r}_1', \omega)f(\mathbf{r}_2', \omega)\right] \otimes \left[\sum_m\sum_n \delta^*(\mathbf{r}_1' - \mathbf{r}_m')\delta(\mathbf{r}_2' - \mathbf{r}_n')\right]\right\rangle. \tag{7}$$

If we assume that the average over the ensemble of each article and that over the ensemble of the distribution to be mutually independent, the correlation function can be expressed as

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) \otimes C_n(\mathbf{r}_1', \mathbf{r}_2', \omega), \tag{8}$$

where

$$C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) = \left\langle f^*(\mathbf{r}_1', \omega)f(\mathbf{r}_2', \omega)\right\rangle \tag{9}$$

is the correlation function of each particle and

$$C_n(\mathbf{r}_1', \mathbf{r}_2', \omega) = \left\langle \sum_m\sum_n \delta^*(\mathbf{r}_1' - \mathbf{r}_m', \omega)\delta(\mathbf{r}_2' - \mathbf{r}_m', \omega)\right\rangle \tag{10}$$

is the correlation of the distribution function of the collection.

We assume that the scattering is weak so that the scattered field can be discussed within the accuracy of the first-order Born approximation [19]. Then the cross-spectral density function of the far-zone scattered field can be expressed as [18]

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{C}_F\left[-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega\right], \tag{11}$$

where

$$\tilde{C}_F(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_F(\mathbf{r}_1', \mathbf{r}_2', \omega)\exp\left[-i(\mathbf{K}_1\cdot\mathbf{r}_1' + \mathbf{K}_2\cdot\mathbf{r}_2')\right]d^3r_1'd^3r_2' \tag{12}$$

is the six-dimensional spatial Fourier transform of the correlation function of the scattering potentials of the whole collection, and

$$\mathbf{K}_1 = -k(\mathbf{s}_1 - \mathbf{s}_0), \quad \mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0). \tag{13}$$

On substituting from Eq. (8) into Eq. (12), after manipulating the Fourier transform, one can obtain the expression for the Fourier transform of the correlation function of the scattering potentials as follows:

$$\tilde{C}_F(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega)\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega), \tag{14}$$

where

$$\tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_f(\mathbf{r}_1', \mathbf{r}_2', \omega)\exp\left[-i(\mathbf{K}_1\cdot\mathbf{r}_1' + \mathbf{K}_2\cdot\mathbf{r}_2')\right]d^3r_1'd^3r_2' \tag{15}$$

is the Fourier transform of the correlation function of the scattering potentials of each particle, and

$$\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_n(\mathbf{r}_1', \mathbf{r}_2', \omega)\exp\left[-i(\mathbf{K}_1\cdot\mathbf{r}_1' + \mathbf{K}_2\cdot\mathbf{r}_2')\right]d^3r_1'd^3r_2' \tag{16}$$

is the Fourier transform of the correlation of the distribution function, which is also known as the pair-structure factor in some previous discussions [15].

3. The spectral degree of coherence of the far-zone scattered field

The spectral degree of coherence of the far-zone scattered field can be obtained from its cross-spectral density function, which is defined as [18]

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega)}{\sqrt{S^{(s)}(r\mathbf{s}_1, \mathbf{s}_0, \omega)}\sqrt{S^{(s)}(r\mathbf{s}_2, \mathbf{s}_0, \omega)}}, \tag{17}$$

where

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = W^{(s)}(r\mathbf{s}, r\mathbf{s}, \mathbf{s}_0, \omega) \tag{18}$$

is the spectral density of the far-zone scattered field. On substituting from Eq. (11) together with Eq. (14) first into Eq. (18), and then into Eq. (17), one can find the spectral degree of coherence of the scattered field, with a form of

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \mu_f^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega)\mu_n^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) \tag{19}$$

where

$$\mu_f^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{C}_f\left[\mathbf{K}_1, \mathbf{K}_2, \omega\right]}{\sqrt{\tilde{C}_f\left[\mathbf{K}_1, -\mathbf{K}_1, \omega\right]}\sqrt{\tilde{C}_f\left[-\mathbf{K}_2, \mathbf{K}_2, \omega\right]}} \tag{20}$$

is the normalized correlation coefficient of the Fourier transform