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Correlation between intensity fluctuations of polychromatic beams generated by quasi-homogeneous sources and the scaling law

Meilan Luo, Daomu Zhao*

Department of Physics, Zhejiang University, Hangzhou 310027, China

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ABSTRACT

The far-zone correlation between intensity fluctuations with polychromatic beams generated by quasi-homogeneous sources is investigated by giving two illustrative examples. The analyses show that, in general, the frequency components of correlation between intensity fluctuations may change on propagation. Moreover, we have discussed the extreme case of critical angle at which no frequency shift occurs, and also proposed the so-called scaling law for correlation between intensity fluctuations.

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1. Introduction

Correlation between intensity fluctuations is of considerable interest, apart from being originally introduced to determine the angular diameters of radio stars [1], as well as many potential applications in various areas [2–4]. Most especially, it is used in the area of ghost imaging as a tool for retrieving an object's transmittance pattern more recently [5–7]. And the measurements of the correlation between intensity fluctuations can be acquired through the well-known Hanbury Brown and Twiss (HBT) experiment, which was conducted for the first time in 1950s [8].

The topic associated with correlation between intensity fluctuations generated by stochastic electromagnetic fields has been studied extensively in connection with propagation through free space [9–11], or scattering by random media [12,13]. However, the sources utilized in the most of previous investigations are confined to be monochromatic light or quasi monochromatic light. Besides, as far as we know, very little attention has been paid to the influence of frequency on the correlation between intensity fluctuations. And only recently has the closely related analysis on the ghost image, which strongly depends on the intensity fluctuations, been made with two wavelengths or multi wavelengths [14,15]. Among which the obtained results show somewhat distinctly from those by quasi monochromatic light, and this underlines the significant effect of the wavelength on the intensity fluctuations to some extent. So it is natural to ask how the correlation between intensity fluctuations will behavior with polychromatic beams. To this end, we examine in this paper theoretically the dependency of the normalized correlation between intensity fluctuations on the source correlation, such as the frequency components and observation directions.

2. Theory

First we begin by reviewing some basic results of the correlation between intensity fluctuations for following analysis. To do this, we consider the fluctuation in intensity of a field $E(\boldsymbol{\rho}, \omega)$ at a point $\boldsymbol{\rho}$ and frequency ω, which is governed by

$$\Delta I(\boldsymbol{\rho}, \omega) = I(\boldsymbol{\rho}, \omega) - \langle I(\boldsymbol{\rho}, \omega)\rangle, \tag{1}$$

where the angular brackets denote the ensemble average, and $I(\boldsymbol{\rho}, \omega)$ means the instantaneous intensity and is calculated from

$$I(\boldsymbol{\rho}, \omega) = E^{*}(\boldsymbol{\rho}, \omega)E(\boldsymbol{\rho}, \omega), \tag{2}$$

here the star denotes the complex conjugate.

Making an assumption that the fluctuations of the field obey the Gaussian statistics, and using the moment theorem for complex Gaussian random process [16], then it follows that correlation between intensity fluctuations at two points takes the form

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \equiv \langle \Delta I(\boldsymbol{\rho}_1, \omega)\Delta I(\boldsymbol{\rho}_2, \omega)\rangle = \left|W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)\right|^2, \tag{3}$$

where $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \left\langle E^{*}(\boldsymbol{\rho}_1, \omega)E(\boldsymbol{\rho}_2, \omega)\right\rangle$ is the cross-spectral density, used to describe the statistical properties of the beam at a pair of points.

* Corresponding author.
E-mail address: zhaodaomu@yahoo.com (D. Zhao).

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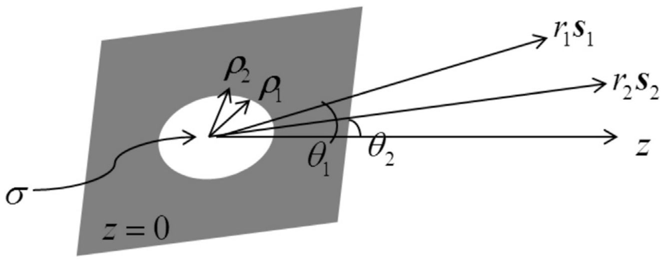

Fig. 1. Illustrating the notation relating to radiation from a planar, secondary quasi-homogeneous source σ.

Next let us consider a planar, secondary, quasi-homogeneous source (see Fig. 1), for which the expression of the cross-spectral density can be approximated by the formula [16]

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \approx S^{(0)}((\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2, \omega)\mu^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega). \tag{4}$$

Recalling that the expression for the cross-spectral density of the far field is derived to be

$$W^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega) = \frac{(2\pi k)^2 \cos\theta_1 \cos\theta_2}{r_1 r_2}\tilde{W}^{(0)}(-k\mathbf{s}_{1\perp}, k\mathbf{s}_{2\perp}, \omega), \tag{5}$$

where the phase term $\exp[ik(r_2 - r_1)]$ along the axis between the two reference planes has been omitted, θ_1 and θ_2 denote the angles which the unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ make with the positive z-axis, with $\mathbf{s}_\perp$ being the projection, $\tilde{W}^{(0)}(-k\mathbf{s}_{1\perp}, k\mathbf{s}_{2\perp}, \omega)$ is the four-dimensional spatial Fourier transform of the cross-spectral density in the source plane, i.e.

$$\begin{aligned}\tilde{W}^{(0)}(\mathbf{f}_1, \mathbf{f}_2, \omega) &= \frac{1}{(2\pi)^4}\iint W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)\exp[-i(\mathbf{f}_1\cdot\boldsymbol{\rho}_1 + \mathbf{f}_2\cdot\boldsymbol{\rho}_2)]d^2\boldsymbol{\rho}_1 d^2\boldsymbol{\rho}_2\\ &= \tilde{S}^{(0)}(\mathbf{f}_1 + \mathbf{f}_2, \omega)\tilde{\mu}^{(0)}((\mathbf{f}_2 - \mathbf{f}_1)/2, \omega),\end{aligned} \tag{6}$$

where $\tilde{S}^{(0)}$ and $\tilde{\mu}^{(0)}$ are the two-dimensional Fourier transforms of $S^{(0)}$ and $\mu^{(0)}$, respectively:

$$\tilde{S}^{(0)}(\mathbf{f}, \omega) = \frac{1}{(2\pi)^2}\int_{z=0} S^{(0)}(\boldsymbol{\rho}, \omega)\exp(-i\mathbf{f}\cdot\boldsymbol{\rho})d^2\rho, \tag{7}$$

$$\tilde{\mu}^{(0)}(\mathbf{f}', \omega) = \frac{1}{(2\pi)^2}\int_{z=0} \mu^{(0)}(\boldsymbol{\rho}', \omega)\exp(-i\mathbf{f}'\cdot\boldsymbol{\rho}')d^2\rho'. \tag{8}$$

On substituting from Eq. (6) into Eq. (5), then it follows immediately that

$$\begin{aligned}W^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega) = \frac{(2\pi k)^2 \cos\theta_1 \cos\theta_2}{r_1 r_2} \times \tilde{S}^{(0)}(k(\mathbf{s}_{2\perp} - \mathbf{s}_{1\perp}), \omega)\\ \tilde{\mu}^{(0)}(k(\mathbf{s}_{2\perp} + \mathbf{s}_{1\perp})/2, \omega).\end{aligned} \tag{9}$$

Finally, after inserting Eq. (9) into Eq. (3) the following expression for the correlation between intensity fluctuations at far field is obtained to be

$$\begin{aligned}C^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega) &= \left[\frac{(2\pi k)^2 \cos\theta_1 \cos\theta_2}{r_1 r_2}\right]^2\\ &\times \left[\tilde{S}^{(0)}(k(\mathbf{s}_{2\perp} - \mathbf{s}_{1\perp}), \omega)\tilde{\mu}^{(0)}(k(\mathbf{s}_{2\perp} + \mathbf{s}_{1\perp})/2, \omega)\right]^2.\end{aligned} \tag{10}$$

This is a general formula by which correlation between intensity fluctuations generated by a quasi-homogeneous source may be discussed. And it is to be noted that the angular dependence of correlation between intensity fluctuations is not only given by the factor $\cos^2\theta_i$, $(i = 1, 2)$, but also dependent on the directional vector $\mathbf{s}$ of the spectral density and the spectral degree of coherence of the source.

3. Two examples

To better illustrate how the relative correlation between intensity fluctuations behaviors after propagation, we proceed with a normalized formalism, which is similar to that describing the spectral changes [17–19]. The normalized correlation between intensity fluctuations is introduced to be

$$C_N^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega) = \frac{C^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega)}{\int_0^\infty C^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega)d\omega}. \tag{11}$$

In view of Eqs. (10) and (11), it is readily observed that $C_N^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega)$ is essentially determined by the square of two-dimensional spatial Fourier transform of the spectral density of the source and the state of coherence across the source. One may have noticed that if taking the dependency of coherence part on frequency into account, the calculation of the integration in Eq. (11) will become too tedious. Hence, we refer to the treatment about spectral changes on propagation [17–19], considering two typical examples in which only the source spectrum depends on frequency.

First a special case is the source with the same spectrum at each point and possessing a Gaussian spectral profile with bandwidth Γ_0 [17,18], it is

$$S^{(0)}(\boldsymbol{\rho}, \omega) \equiv S^{(0)}(\omega) = \exp\left[-\frac{(\omega - \omega_0)^2}{2\Gamma_0^2}\right]. \tag{12}$$

The part of the degree of coherence across the source plane also takes a Gaussian form

$$\mu^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega) = \exp\left[-\frac{(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2}{2\delta^2}\right], \tag{13}$$

where δ measures the coherence length, all these parameters are effectively independent of the frequency.

On substituting from Eqs. (7), (8), (10), (12) and (13) into the formula (11), and using a more compact form

$$\frac{(\omega - \omega_0)^2}{2\Gamma_0^2} + \frac{\delta^2(\sin\theta_1 + \sin\theta_2)^2}{4} = \alpha_1^2(\omega - \omega'_0)^2 + \beta^2, \tag{14}$$

with $\alpha_1^2 = \frac{1}{\Gamma_0^2} + \frac{\delta^2(\sin\theta_1 + \sin\theta_2)^2}{4c^2}$, $\omega'_0 = \frac{\omega_0}{\alpha^2\Gamma_0^2}$, $\beta^2 = \frac{\omega_0^2\delta^2(\sin\theta_1 + \sin\theta_2)^2}{4c^2\alpha^2\Gamma_0^2}$.

One can obtain the expression for the far-zone normalized correlation between intensity fluctuations

$$\begin{aligned}C_N^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega) &= \frac{\omega^4[S^{(0)}(\omega)]^2\exp[-\delta^2(\sin\theta_1 + \sin\theta_2)^2k^2/4]}{\int_0^\infty \omega^4[S^{(0)}(\omega)]^2\exp[-\delta^2(\sin\theta_1 + \sin\theta_2)^2k^2/4]d\omega}\\ &= \frac{\alpha_1\omega^4}{\sqrt{\pi}\left(\omega'^4_0 + \frac{3\omega'^2_0}{\alpha_1^2} + \frac{3}{4\alpha_1^4}\right)}\exp[-\alpha_1^2(\omega - \omega'_0)^2].\end{aligned} \tag{15}$$

On the above evaluation, for convenience we have assumed $\Gamma_0/\omega_0 \ll 1$, which means the rms width of the Gaussian distribution of the above integration is much smaller than ω_0, under this circumstance the lower zero limit could be replaced by $-\infty$, at a good approximation.

From Eq. (15) one may notice that, different form the normalized correlation between intensity fluctuations of the source, which keeps the same at each point, $C_N^{(\infty)}(r_1\mathbf{s}_1, r_2\mathbf{s}_2, \omega)$ depends on the direction of observation. And only when $\theta_1 = \theta_2 = 0$, i.e., for on-axis points, $\omega'_0 = \omega_0$ holds. Otherwise, the Gaussian distribution is centered at a lower frequency than the Gaussian distribution that

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