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Far-field intensity of Lorentz related beams

Xi Peng [a,b], Chidao Chen [a,b], Bo Chen [a,b], Yulian Peng [a,b], Meiling Zhou [a,b], Liping Zhang [a,b], Dongdong Li [a,b], Dongmei Deng [a,b,*]

[a] Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices, South China Normal University, Guangzhou 510631, China
[b] CAS Key Laboratory of Geospace Environment, University of Science & Technology of China, Chinese Academy of Sciences, Hefei 230026, China

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ABSTRACT

We introduce a sufficient condition under which the Lorentz beam convolution with other beams constitutes valid cross-spectral densities. Two examples are given to show how the Lorentz related beam can be used for generation of a far field being a modulated version of another one. The far-field intensity patterns in the Cartesian symmetries by the convolution operation of the Lorentz beams with multi-sinc beams, and the convolution operation of the Lorentz beams with multi-sinc Gaussian beams, are shown respectively. We find that different beam order can result distinct far field changes.

1. Introduction

The research of Lorentz beams is particularly stimulating in the present case because of the physical realizability of the proposed field since Gawhary and Severini first introduced this new kind of beam [1,2]. This realizability is from a practical point of view [3–7] that certain laser sources produce fields which shows fundamental variations with respect to the canonical Gaussian beam [8–10]. The transverse pattern of the Lorentz beam in the source plane is the product of two independent Lorentz functions, provides an appropriate model to describe some certain laser sources that produce a relatively high divergent field, which has been under intensive research recently [11,12]. It is also an appropriate model to describe some certain laser phenomenon, Zhou derived the analytical TE and TM terms of a Lorentz-Gauss beam in the far field according to the vectorial structure of electromagnetic beam and the method of stationary phase [3], the analytical propagation equation of a nonparaxial Lorentz-Gauss beam in free space on the basis of vectorial Rayleigh-Sommerfeld integrals formulae [6], the propagation equation of a partially coherent Lorentz-Gauss beam through a paraxial and real ABCD optical system [12].

Recently, partially coherent sources in the far field have long been a subject of great importance in a broad area of physical optics, and have received much attention due to they constitute new forms of spatial correlation functions [13–21]. And Fei Wang et al. reported the experimental generation of partially coherent beams with different complex degrees of coherence [22]. Certain integrability conditions with respect to spatial and spectral variables, the quasi-Hermiticity and the non-negative definiteness are included to satisfy the conditions of the cross spectral density (CSD) [23,24]. This provided possibilities for achieving far-field intensity distributions by generating new beam sources, such as the multi-Gaussian Schell-model and sinc-Schell model sources producing tunable flat profiles [17], the multi-sinc beams producing adjustable multi-rings and lattice patterns [19], the convolution of the Gaussian beam and sinc beams producing filtered beams [23]. Korotkova and Mei have recently confirmed that the convolution of two degrees of coherence represents the novel legitimate degree of coherence for Schell like sources [24]. They have also explored the possibility of assigning to the different elements of the CSD matrix the profiles belonging to different families of functions [25].

However, to the best of our knowledge, the Lorentz beam convolution with other beams has not mentioned yet. The purpose of this paper is to derive the inherent relationships between far-field beam pattern properties of the convolution of Lorentz related beams. We choose two legitimate degrees of coherence in the source plane based on the weighted superposition rule. For illustration of these relationships, we show useful far-field intensity patterns in the Cartesian symmetries by the convolution operation of the Lorentz beams with multi-sinc beams, and the convolution operation of the Lorentz beams with multi-sinc Gaussian beams, respectively. Compared to the work about Gaussian beam convolution with sinc beam in Ref. [23], Lorentz related beams have shown distinct far field intensity characteristics. By choosing beam

* Corresponding author at: Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices, South China Normal University, Guangzhou 510631, China.
E-mail address: dmdeng@263.net (D. Deng).

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order N odd or even, the far-field intensity shape can be modulated. Some useful and interesting results are found in our investigations.

2. Theory

We set the beam at a pair of points with $\mathbf{r}_1(x_1, y_1)$ and $\mathbf{r}_2(x_2, y_2)$, the CSD of a light field of a planar source will be denoted by $W(\mathbf{r}_1, \mathbf{r}_2)$, the explicit dependence on the temporal frequency is omitted here. According to the superposition rule, the sufficient condition for the CSD to be genuine is that it can be expressed as the integral [15]

$$W(\mathbf{r}_1', \mathbf{r}_2', z) = \iint_{-\infty}^{\infty} P(\nu)H^*(\mathbf{r}_1', z, \nu)H(\mathbf{r}_2', z, \nu)d^2\nu, \tag{1}$$

where $P(\nu)$ should be a nonnegative function [18], and

$$H^*(\mathbf{r}_1', z, \nu)H(\mathbf{r}_2', z, \nu) = \left(\frac{k}{2\pi z}\right)^2 \iint_{-\infty}^{\infty} H_0^*(\mathbf{r}_1, \nu)H_0(\mathbf{r}_2, \nu) \times e^{-\frac{ik}{2z}\left[(\mathbf{r}_1'-\mathbf{r}_1)^2-(\mathbf{r}_2'-\mathbf{r}_2)^2\right]}d^2\mathbf{r}_1 d^2\mathbf{r}_2, \tag{2}$$

we consider a Fourier-like structure for H_0

$$H_0(\mathbf{r}, \nu) = e^{-\frac{\mathbf{r}^2}{2\sigma^2}}e^{-2\pi i\nu\cdot\mathbf{r}}, \tag{3}$$

where σ is the typical beam side.

In the Cartesian symmetry case, the CSD in the source plane can be expressed as a factorized form

$$W(\mathbf{r}_1, \mathbf{r}_2) = \prod_{l=x,y} W(l_1, l_2), \tag{4}$$

where

$$W(l_1, l_2) = \int_{-\infty}^{\infty} P(\nu_l)H_0^*(l_1, \nu_l)H_0(l_2, \nu_l)d\nu_l. \tag{5}$$

When $\mathbf{r}_1' = \mathbf{r}_2' = \mathbf{r}'$, we obtain the spectral intensity of the partially coherent field at the half-space $z > 0$, partially coherent beams are often assumed to be Shell-model beams [20], it can be expressed as the product of 1D integral representations

$$S(\mathbf{r}', z) = \prod_{l'=x',y'} \int_{-\infty}^{\infty} P(\nu_l)|H_l(l', z, \nu_l)|^2 d\nu_l, \tag{6}$$

where

$$|H_l(l', z, \nu_l)|^2 = \frac{\sigma}{w(z)}e^{-(l'+2\pi z\nu_l/k)^2/w^2(z)}, \tag{7}$$

$$w^2(z) = \sigma^2 + z^2/(k\sigma)^2, \tag{8}$$

and $P(\nu_l)$ should be a nonnegative function [18], determined by the source degree of coherence $\mu(l)$

$$p(\nu_l) = \int_{-\infty}^{\infty} \mu(l)e^{-2\pi i\nu_l\cdot l}dl. \tag{9}$$

3. Analysis and discussion

Now, let us consider the source degree of coherence μ that is the convolution of two legitimate degrees of coherence.

3.1. The Lorentz beam and multi-sinc beams

First we will introduce the convolution of the Lorentz beam and multi-sinc beams. The degree of coherence of Lorentz beams can be written as

$$\mu_L(\mathbf{r}_1 - \mathbf{r}_2) = A\prod_{l=x,y} \frac{1}{w_0}\frac{1}{1 + \left(\frac{l_1 - l_2}{w_0}\right)^2}, \tag{10}$$

where A is a normalized factor, the degree of coherence of multi-sinc beams can be written as

$$\mu_S(\mathbf{r}_1 - \mathbf{r}_2) = \prod_{l=x,y} \frac{1}{B}\sum_{n=1}^{N} \frac{(-1)^{n-1}}{C_n}\mathrm{sinc}\left(\frac{l_1 - l_2}{C_n d}\right), \tag{11}$$

where $B = \sum_{n=1}^{N} \frac{(-1)^n}{C_n}$ is the normalization factor, N means different orders, $C_n = \sqrt[m]{(2N-1)/[2^m(2N-2n+1)]}$, m is an arbitrary positive real number and we choose $m=1.5$ here, sinc(x) is the normalized sinc function, and d is the correlation width.

The new degree of coherence can be written as

$$\mu_1(\mathbf{r}_1 - \mathbf{r}_2) = \mu_L(\mathbf{r}_1 - \mathbf{r}_2) \otimes \mu_S(\mathbf{r}_1 - \mathbf{r}_2) = A\prod_{l=x,y} \frac{1}{w_0}\frac{1}{1 + \left(\frac{l_1 - l_2}{w_0}\right)^2} \otimes \frac{1}{B}\sum_{n=1}^{N} \frac{(-1)^{n-1}}{C_n}\mathrm{sinc}\left(\frac{l_1 - l_2}{C_n d}\right), \tag{12}$$

where $\otimes$ means the convolution.

Then we find that the Fourier transform of the new degree of coherence can be expressed as the product of two nonnegative function, the kernel $P_1(\nu)$ has the form

$$P_1(\nu) = P_L(\nu)P_S(\nu) = A\prod_{l=x,y} \frac{\pi d}{B}\sum_{n=1}^{N} (-1)^{n-1}rect(C_n d\nu_l)\exp[-2\pi|\nu_l|w_0], \tag{13}$$

where P_L is the nonnegative weight function of Lorentz beams, P_S is the nonnegative weight function of multi-sinc beams, and *rect* (x) is the rectangular function taking on value 1 for $|x| < 0.5$ and 0 otherwise. In this paper we choose $\lambda = 632.8$ nm, $w_0 = 100\lambda$, $\sigma = 1$ mm, $d = 0.1$ mm.

In Fig. 1(c) and (f), there are solid lines for convolution of multi-sinc beams with the Lorentz beam while red dotted lines for convolution of multi-sinc beams with the Gaussian beam. It is clearly seen from Fig. 1 that the shape between two kinds of the convolution is same, but the profile details have differences, more information are shown in Fig. 3. Fig. 2 illustrates the nonnegative weighting functions P_L, P_S and P_1 in solid line, the shape of the weighting function P_1 keeps the weighting function profile of multi-sinc beams but modified by weighting function profile of the Lorentz beam.

From Eqs. (6) and (13), we can obtain the far-field spectral density behavior in Fig. 3 with the Cartesian symmetry, the modified multi-sinc beams are filtered by a soft-edge Lorentz aperture and shape the rectangular lattice patterns with the Lorentz envelop, which is different from the Fig. 7 in Ref. [23], the Lorentz shape in center makes the beam distinct. For same N_x and N_y, the center graph is an Lorentz filtered multi-sinc beam model when N is odd, four uniform parts of Lorentz filtered multi-sinc beams when N is even. By choosing different N_x and N_y, one can modulate the center graph depart into two part in one direction, x direction or y direction, respectively. Hence, in such case, by setting different N_x and N_y, we can make far-field intensity shape of novel Lorentz related sources in different forms.

3.2. The Lorentz beam and multi-sinc Gaussian beams

Let us now consider the convolution of the Lorentz beam and multi-sinc Gaussian beams. The degree of coherence of multi-sinc Gaussian beams can be written as