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Stitching error calculation method for the slope data on a square subaperture

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The data on subaperture in this paper is collected by a Hartmann test device or a Hartmann–Shack wavefront sensor. The slope data on each subaperture of the test optical component is obtained via the linear least squares fitting routine in this paper. Importantly, the error transfer formulas are deduced when the measurement data are polluted by the random noise, which will affect the uncertainty of stitching parameters, such as tilt or defocus. In particular, the accuracy formulas for stitching evaluation, which can be used to calculate the stitching error of each point on each subaperture for both the parallel mode and the serial mode, are derived mathematically. Therefore, this paper provides these formulas to estimate exactly the stitching error in each subaperture if the variance of random noise has been estimated and if the overlapping area is also given in advance. The results of simulation experiments show that the stitching accuracy formulas proposed are verified and they can be used to evaluate stitching accuracy. The reason why the error of the parallel stitching mode is less than that of the serial stitching mode is presented theoretically.

1. Introduction

With the development of science and technology, large optical components are widely used in space telescopes [1,2], laser fusion systems [3,4] and other aspects. In traditional optical measurements, the large optical surface is tested by a large-scale interferometer. However, the interference testing method requires a flat reference, and most interferometers are sensitive to the measurement environment. As a result, the cost of a large-scale interferometer is greatly increased, and good environment control is required for testing [5,6]. In 1982, the subaperture measuring method was first introduced in interferometric metrology by C.J. Kim at the Optical Sciences Center at the University of Arizona, [7]. Since the method was proposed, it has been widely used in the measurement of large aperture optical elements. More recently, subaperture stitching interferometry is becoming increasingly popular. In 1994, Otsubo proposed a global subaperture stitching method based on stitching parameters; in this method, the misalignment errors is simultaneously fitted and obtained over multiple overlapping subapertures [8]. In 2003, QED Technologies successfully developed an automated subaperture stitching interferometer. This interferometer can successfully test the surface of flat and spherical optical elements with diameters of less than 200 mm [9]. Moreover, research studies have also focused on the analysis of the accuracy of subaperture stitching. To date, there are three types of methods to evaluate the accuracy of subaperture stitching technique. In the first type, the stitching results are compared with the results of the full aperture interferometry; this method of evaluation is the most simple and straightforward one [10]. In the second type, the effect of random noise and overlapping area on the stitching parameters is qualitatively analyzed. This method can only obtain the error of the stitching parameters between two adjacent subapertures and it cannot obtain the error of each point on each subaperture after the subaperture stitching procedure is completed [8,11]. In the third type, according to the knowledge of statistics, the proportion of the regression sum of squares of the phase difference to the total sum of squares of the phase difference is used to evaluate the accuracy of the stitching [12].

Different from the traditional subaperture stitching method based on wavefront measurements, the subaperture wavefront slope stitching test is based on a Hartmann–Shack wavefront sensor [13]. Hartmann–Shack sensors have the advantages of insensitivity to the air disturbance and vibration, rapid testing and high dynamic range. In addition, subaperture wavefront slope stitching tests can avoid the influence of the piston. Thus, this method has a unique advantage in the testing of a large aperture optical element. Due to these characteristics, an increasing number of scholars are studying the subaperture wavefront slope stitching testing method [14–16]. Currently, research studies on

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wavefront slope stitching are mainly focused on the establishment of the mathematical mode and qualitative analysis of the stitching error. However, no mathematical formula is available to evaluate stitching accuracy when random noise exists in measurement process. In this paper, we describe in detail the following: 1) the effect of random noise and overlapping area on the stitching parameters and 2) the error relationship between the stitching parameters and the adjusted slope. Importantly, the error transfer formula for the slope stitching is first determined. On this basis, the respective formulas for solving the accuracy of serial slope stitching mode and parallel slope stitching mode are derived. This paper shows that these formulas are verified by our simulation experiments. The RMS and PV values of the residual error for the reconstruction of the wavefront based on a given slope matrix in our simulation experiments demonstrate that the error of parallel stitching is statistically less than that of serial stitching.

2. Principles

As mentioned in the introduction, a large-size component can be measured using the stitching method. In this manner, we can obtain a wider range of measurement area with good resolution using a HS with a limited aperture. The disadvantages of the stitching methods include error propagation, and we must move the sensor or the sample repeatedly to cover the entire surface.

Therefore, the slope data on a large aperture consists of multiple slope matrices on different sub-regions. Due to the misalignment, the piston, tilt, and even defocus, will be inevitably introduced into the measurement data when we move the sensor from one place to another. To eliminate the interference of misalignment, the slope data on the overlapping area between two adjacent subapertures are used to obtain the corresponding stitching parameters by a linear least squares fitting routine. As a result, these parameters, such as coefficient of tilt or defocus, are obtained, and then this procedure is repeated to the end of the stitching task until all these parameters are obtained. Next, these parameters are treated to construct the stitched slope data on the whole component. For simplicity of discussion, as shown in Fig. 1, we assume that the two adjacent subapertures W_1 and W_2 have a common area due to their overlapping area, and the measured phase distributions over the two areas are denoted as Φ_1 and Φ_2, respectively. The connection of the two adjacent areas is considered first.

Let W_1 be the reference plane, and let the relationship between the phase distributions Φ_1 and Φ_2 in the common area fulfill the following equation:

$$\Phi_2 = \Phi_1 + P + T_x x + T_y y, \tag{1}$$

where P is the piston, and T_x and T_y are the coefficients of the tilt in the x and y directions, respectively. Considering the derivation of x-tilt and y-tilt in Eq. (1), we have

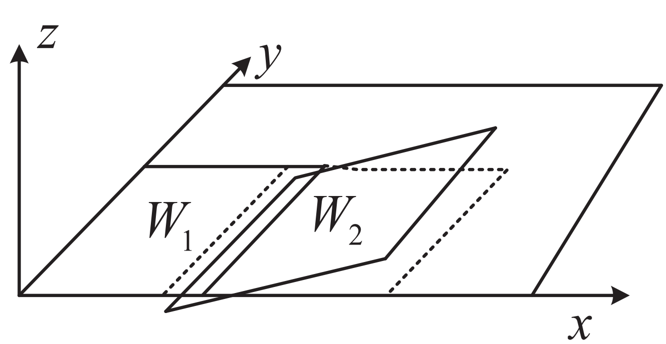

Fig. 1. Schematic of subaperture stitching.

$$\begin{aligned} G_{2x} - G_{1x} &= T_x \\ G_{2y} - G_{1y} &= T_y, \end{aligned} \tag{2}$$

where $G_{1x} = \partial\Phi_1/\partial x$ and $G_{1y} = \partial\Phi_1/\partial y$ are the x-tilt and y-tilt measured slopes, respectively, over the area W_1, and $G_{2x} = \partial\Phi_2/\partial x$ and $G_{2y} = \partial\Phi_2/\partial y$ are the x-tilt and y-tilt measured slopes over the area W_2, respectively. The parameter P disappears in Eq. (2) after taking the derivative. In other words, the piston does not influence the measured slope on subapertures; thus, we only need to find the coefficient T_x of x-tilt and coefficient T_y of y-tilt. In general, a minimum number of sampling points in the common area are chosen to accomplish the calculation of the values of T_x and T_y within an acceptable stitching error because the existence of random noise during measurement will affect the calculation accuracy of parameters.

Suppose there are n sampling points in their common area; in this case, we can obtain the corresponding equations from Eq. (2) as follows:

$$\begin{bmatrix} G_{2x}(x_1 y_1) - G_{1x}(x_1, y_1) \\ G_{2x}(x_2 y_2) - G_{1x}(x_2, y_2) \\ \cdots \\ G_{2x}(x_n, y_n) - G_{1x}(x_n, y_n) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} [T_x], \tag{3a}$$

$$\begin{bmatrix} G_{2y}(x_1 y_1) - G_{1y}(x_1, y_1) \\ G_{2y}(x_2 y_2) - G_{1y}(x_2, y_2) \\ \cdots \\ G_{2y}(x_n, y_n) - G_{1y}(x_n, y_n) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} [T_y]. \tag{3b}$$

Accordingly, T_x and T_y are the averages of the differences in the left-hand side of Eqs. (3a) and (3b). Next, the adjusted results of G_{2x} and G_{2y} can be solved by using Eq. (2). Subsequently, the adjusted results of G_{2x} and G_{2y} on the W_2 can be connected with the results of G_{1x} and G_{1y} on the W_1 successfully; as a result, we obtain the stitched slope on the two adjacent areas.

Next, consider another case. Both tilt and defocus are introduced in the measured slope data, and the relationship between the phase distributions Φ_1 and Φ_2 in their common area fulfills the following equation:

$$\Phi_2 - \Phi_1 = P + T_x x + T_y y + D(x^2 + y^2), \tag{4}$$

where D is the coefficient of defocus. Likewise, considering the derivation of x-tilt and y-tilt in Eq. (4), we have

$$\begin{aligned} G_{2x} - G_{1x} &= T_x + 2D_x x \\ G_{2y} - G_{1y} &= T_y + 2D_y y. \end{aligned} \tag{5}$$

Similarly, suppose there are n sampling points in the common area. From Eq. (5), we obtain

$$\begin{bmatrix} G_{2x}(x_1 y_1) - G_{1x}(x_1, y_1) \\ G_{2x}(x_2 y_2) - G_{1x}(x_2, y_2) \\ \cdots \\ G_{2x}(x_n, y_n) - G_{1x}(x_n, y_n) \end{bmatrix} = \begin{bmatrix} 1, x_1 \\ 1, x_2 \\ \vdots \\ 1, x_n \end{bmatrix} \begin{bmatrix} T_x \\ 2D_x \end{bmatrix}, \tag{6a}$$

$$\begin{bmatrix} G_{2y}(x_1 y_1) - G_{1y}(x_1, y_1) \\ G_{2y}(x_2 y_2) - G_{1y}(x_2, y_2) \\ \cdots \\ G_{2y}(x_n, y_n) - G_{1y}(x_n, y_n) \end{bmatrix} = \begin{bmatrix} 1, y_1 \\ 1, y_2 \\ \vdots \\ 1, y_n \end{bmatrix} \begin{bmatrix} T_y \\ 2D_y \end{bmatrix}. \tag{6b}$$

According to the least-squares method, we obtain the coefficients of T_x, T_y and D_x, D_y in Eq. (6a) and (6b). Next, the adjusted results of G_{2x} and G_{2y} can be obtained by using Eq. (5). As a result, the stitched slope on the two adjacent areas is obtained.

3. Error analysis

For an actual measurement, the slope data measured by a